Mathematics I – academic year 2016/17
(preliminary) plan of lectures

week

1. Vector space, linearly dependent and linearly independent vectors, dimension, basis, subspace. Special spaces R^n, E_n and $V(E_n)$. Operations with vectors in $V(E_n)$ (addition, subtraction, multiplication of a vector by a real number, scalar product of two vectors). Matrix of the type $m \times n$, matrices: transposed, upper triangular, square, identity.

2. Equality of two matrices, operations with matrices (the sum, difference, multiplication by a real number, multiplication of two matrices). Rank of a matrix. How to find the rank of a matrix.

   Determinant of a square matrix. Properties and calculation of determinants. Regular and singular matrix. Inverse matrix, Conditions for the existence and calculation of the inverse matrix.

3. System of linear algebraic equations, its matrix form. Gauss' elimination. Frobenius' theorem. Existence and number of solutions of homogeneous and inhomogeneous systems, the structure of the set of all solutions.
   Cramer's rule. Eigenvalue and eigenvector of a square matrix, geometrical interpretation.. Characteristic equation of a square matrix. Calculation of eigenvalues and eigenvectors of a concrete 2×2 and 3×3 matrices.

4. Set of real numbers R, extended set of real numbers R^*, operations and ordering in set R^*. Types of neighbourhoods of points in R^*. A sequence of real numbers, sequences: upper bounded, lower bounded, bounded, increasing, decreasing. A subsequence.
   Limit of a sequence. Basic theorems on limits of sequences, calculation of limits of simple sequences.

5. *Survey and completion of notions from elementary mathematics:* Function of one real variable, domain of definition, range, graph. Restriction of a function. Function even, odd, periodic. Composite function. Inverse function. Function upper bounded, lower bounded, bounded, increasing, decreasing, non–increasing, non–decreasing, monotone, strictly monotone. *Survey of basic elementary functions:* power function, polynomials, exponential, logarithmic, trigonometric, inverse trigonometric.
   Limit of a function (finite and infinite, at a finite point or at infinity). One–sided limits. Basic theorems on limits. Calculation of simple limits.
   Continuity of a function (at a point or on an interval), right and left continuity. Theorems on continuity of a sum, difference, product and ratio of two functions. Continuity of a composite function and an inverse function. Darboux' theorem and theorem on existence of extreme values of a continuous function on a bounded closed interval.

6. Derivative of a function at point x_0, one–sided derivatives, improper derivative. Geometric and physical interpretation of derivative. Equation of the tangent line to the graph of a function $y = f(x)$ at the point $[x_0, f(x_0)]$. Approximate solution of the equation $f(x) = 0$ (informatively).
   Existence of derivative implies continuity.
   Derivative of a sum, difference, product, fraction. Derivatives of elementary functions.
   Derivative of a composite function (the chain rule) and an inverse function. Applications to simple examples.
   Higher order derivatives.
   Differential of a function at a point, its geometrical sense, application to approximate calculation of function values.
   The mean value theorem, geometrical meaning. L'Hospital's rule.

7. Consequences of the sign of the derivative for the behaviour of a function on an interval. Local extremes of a function, application of the first and the second derivative. How to find local extremes.
   Global extremes of a continuous function on an interval.

8. Functions concave up and concave down. Point of inflection. Application of the second derivative. How to find points of inflection.
   Asymptotes. Investigation of the behaviour of a function, sketching the graph.

9. Curvature, osculation circle.
   Taylor's polynomial (MacLaurin's polynomial) of the n–th degree of function f at point x_0. Coefficients of Taylor's polynomial. Taylor's theorem, Lagrange's form of the reminder. Approximation of simple functions by Taylor's polynomials.
   Antiderivative, indefinite integral. Existence of an antiderivative and an indefinite integral on an interval. Table of basic indefinite integrals. Integration by parts.

10. Integration by substitution. Application to simple examples.

    Rational functions – introduction. Integration of a rational function with polynomial of at most 3rd degree in the denominator.

11. Integration of functions of the type $\sin^m x \cdot \cos^n x$.

    Integration of irrational functions of the type $R(x,\, \sqrt[n]{(ax+b)/(cx+d)})$.

    Riemann's integral, geometrical and physical interpretation, basic properties.
    Newton–Leibniz' formula.
    Integration by parts in Riemann's integral.

12. Integration by substitution in Riemann's integral.

    Geometrical applications of Riemann's integral: area of a surface, volume of a circular body, length of a curve.
    Riemann's integral as a function of the upper limit.
    Improper Riemann's integral.

Mathematics I – academic year 2016/17
(preliminary) plan of tutorials

week

1. First information on the course.
   Survey of selected important topics from elementary mathematics. Simpler equations and inequalities. Power function, linear function, quadratic function, n–th root, exponential and logarithmic functions, trigonometric functions, domains, graphs.
2. Vectors in E_2 and in E_3, their geometrical interpretation. Linearly dependent and independent vectors. Dimension and basis of a vector space. Operations with matrices. Problems: 2, 8, 15, 22, 25, 42–45, 51, 53, 68, 69, 70, 79, 109, 112, 117, 124, 132, 139.
3. Rank of a matrix. Inverse matrix. Determinants. Problems with parameters. Problems: 143, 148, 149, 160, 161, 166, 168, 171, 174, 176, 177, 180, 185, 190, 200.
4. Systems of linear algebraic equations (homogeneous, inhomogeneous). Problems with parameters. Frobenius' theorem. Cramer's rule. Geometrical interpretation (relative positions of planes and straight lines). Problems: 273, 275, 278, 280, 288, 295, 324, 328, 332, 338, 348, 359, 360.
5. Eigenvalues and eigenvectors of square matrices. Problems: 236, 237, 241, 243, 245.
6. Sequences of real numbers and their limits. Elementary functions (domains, continuity, graphs, etc.) Odd, even functions. Inverse trigonometric functions. Limit of a function. Limit of a composite function. Problems: 575, 577–581, 591, 594, 596, 609, 612, 619–623, 629, 674, 675, 687, 694, 698, 718, 767, 791, 792, 796, 797, 807, 816, 839, 863–865, 902, 909, 929, 931, 932, 937, 939, 943.
7. Derivative of a function. Geometrical and physical interpretation. Equation of a tangent line and a normal line to the graph of a function. Derivative of a composite function (the chain rule). L'Hospital's rule. Problems: 837, 840, 859, 866, 869, 882, 891, 904, 960, 968, 972, 976, 979, 980, 990, 991, 993, 999, 1001, 1003, 1008, 1109, 1110, 1118, 1119, 1017-1020, 1027, 1028, 1062, 1066, 1071.
8. Intervals of monotonicity and local extremes of a function. Global extremes. Problems: 1144–1146, 1151, 1156, 1160, 1209, 1211, 1164, 1169, 1174, 1175, 1177, 1179, 1204, 1211, 1241.
9. Intervals of concavity of a function. Points of inflection. Asymptotes. Behaviour of a function. Problems: 1255, 1256, 1260, 1262, 1264, 1267, 1270, 1274, 1275, 1277, 1278, 1279, 1282, 1292, 1293, 1295, 1308, 1319, 1321.
10. Approximation of functions by Taylor's polynomials. Indefinite integrals – application of the table of basic indefinite integrals, integration by parts. Problems: 1330, 1331, 1333, 1337, 1346, 1351, 1352, 1354, 1362, 1376, 1450, 1452, 1454, 1455, 1459, 1461, 1467, 1473, 1481–1486, 1494, 1504, 1506, 1510.
11. Indefinite integrals: integration by substitution, integration of rational functions. Problems: 1514–1519, 1532–1534, 1542, 1546, 1555, 1572, 1520, 1628, 1720, 1724, 1731, 1733, 1734, 1748, 1749, 1751, 1754, 1755.
12. Integration of functions of the type $\sin^m x \cdot \cos^n x$ and irrational functions of the type $R(x, \sqrt[n]{(ax+b)/(cx+d)})$. Riemann's integral and its evaluation. Newton–Leibniz formula. Integration by parts in Riemann's integral. Problems: 1815, 1823, 1828, 1832, 1892, 1896, 1898, 1985, 1986, 1989, 1991–1993, 1996, 2000, 2002.
13. Integration by substitution in Riemann's integral. Application of Riemann's integral: area of a surface, volume of a circular body, length of a curve. Improper Riemann's integral. Problems: 2010–2012, 2015, 2-17, 2020, 2024, 2030–2032, 2036, 2038–2040, 2044, 2050, 2051, 2054, 2056–2058, 2060, 2063, 2067–2070, 2074, 2075, 2077 + further problems 1–15.

Mathematics I – academic year 2016/17
assessments and exams

Tutorials, assessments: Tutorials are obligatory. Assessment from tutorials (written in the study record) confirms student's presence and activity at the tutorials and elaboration of homework and tests. Assessment is a necessary condition for the exam. (I.e. student can make the exam only with the assessment written in the study record.)

The assessments are written in the last semestral week, not later than one week after. Exceptions are possible only with the explicit agreement of the chair of the institute.

Exam: Students can choose between the levels A (higher) or B (lower), not later than 2 days before the exam. The exam has a written form. Students are supposed to know and understand notions named in the plan of lectures, to know and understand named theorems (including their assumptions) and to be able to apply the theorems to simple problems. Students are recommended to solve individually problems from exam tests from previous years. The level of these problems corresponds to the exam of level A. Material required for the exam of level A coincides with the contents of lectures (without the 13th week) and with the contents of tutorials. The difference between the exams of levels A and B is especially in the choice and complexity of problems solved in the exam test.

Advantage of exam level A: The exam of level A provides by two more credits than the exam of level B. Students, who finish the named courses (exact information on the list of these courses is provided by the study department) with the exam of level A, can complete the bachelor programme already after three years (in an individual study programme) and they are accepted to the master programme without entrance exams.

Basic literature:

[1] J.Neustupa: **Mathematics I.** Czech Technical University, Praha 2004.

[2] J.Neustupa, S.Kračmar: **Problems in Mathematics I.** Czech Technical University, Praha 1999.

[3] **Selected problems from textbook [2].** pdf file

[4] **Mathematics I -** some problems from exam tests in the previous years. pdf file