Czech Technical University in Prague
Faculty of Mechanical Engineering
Department of Technical Mathematics, Karlovo nám. 13, 121 35 Prague 2

Mathematics I, level Alpha – Requirements for exams in academic year 2019/20

1) Vector space, linearly dependent and linearly independent vectors, dimension, basis, subspace. Special spaces $\mathbb{R}^n$, $\mathbb{E}_n$ and $V(\mathbb{E}_n)$. Operations with vectors in $V(\mathbb{E}_n)$: addition, subtraction, multiplication of a vector by a scalar, product (scalar and vector) of two vectors.

Matrix of the type $m \times n$, matrices: transposed, upper triangular, square, identity.

2) Equality of two matrices, operations with matrices (the sum, difference, multiplication by a scalar, multiplication of two matrices). Rank of a matrix. How to find the rank of a matrix.

Determinant of a square matrix. Properties and calculation of determinants. Regular and singular matrix. Inverse matrix, Conditions for the existence and calculation of the inverse matrix.

3) System of linear algebraic equations, its matrix form. Gauss' elimination. Frobenius' theorem. Existence and uniqueness of solutions of homogeneous and non-homogeneous systems, the structure of the set of all solutions.

Cramer's rule. Eigenvalue and eigenvector of a square matrix, geometrical interpretation. Characteristic equation of a square matrix. Calculation of eigenvalues and eigenvectors of 2×2 and 3×3 matrices.

4) Set of real numbers $\mathbb{R}$, extended set of real numbers $\mathbb{R}^*$, operations and ordering in set $\mathbb{R}^*$. Types of neighbourhoods of points in $\mathbb{R}^*$. A sequence of real numbers, properties of sequences: upper bounded, lower bounded, bounded, increasing, decreasing. A subsequence.

Limit of a sequence. Basic theorems on limits of sequences, calculation of limits of simple sequences.

5) *Survey and completion of notions from elementary mathematics:* Function of one real variable, domain of definition, range, graph. Restriction of a function. Function even, odd, periodic. Composite function. Inverse function. Function upper bounded, lower bounded, bounded, increasing, decreasing, non–increasing, non–decreasing, monotone, strictly monotone. *Survey of basic elementary functions:* power function, polynomials, exponential, logarithmic, trigonometric, inverse trigonometric.

Limit of a function (finite and infinite, at a finite point or at infinity). One–sided limits. Basic theorems on limits. Calculation of simple limits.

Continuity of a function (at a point or on an interval), right and left continuity. Theorems on continuity of a sum, difference, product and ratio of two functions. Continuity of a composite function and an inverse function. Darboux's theorem and theorem on existence of extreme values of a continuous function on a bounded closed interval.

6) Derivative of a function at a point, one–sided derivatives, improper derivative. Geometrical and physical interpretation of derivative. Equation of the tangent line to the graph of a function $y = f(x)$ at given point. Differential of a function at a point, its geometrical sense, application to approximate calculation of function values. Approximate solution of the equation $f(x) = 0$ (informatively).

Derivative of a sum, difference, product, fraction. Derivatives of elementary functions. Derivative of a composite function (the chain rule) and an inverse function. Applications to simple examples.
Higher order derivatives.

The mean value theorem, geometrical meaning. L'Hospital's rule.
Existence of derivative implies continuity.

7) Consequences of the sign of the derivative for the behaviour of a function on an interval.

Local extrema of a function, application of the first and the second derivative. How to find local extrema.

Global extrema of a continuous function on an interval.

8) Functions concave up (convex) and concave down (concave). Point of inflection. Application of the second derivative. How to find points of inflection.

Asymptotes. Investigation of the behaviour of a function, sketching the graph.

9) Curvature, osculation circle. Taylor's polynomial (MacLaurin's polynomial) of the n–th degree of function f at a point. Coefficients of Taylor's polynomial. Taylor's theorem, Lagrange's form of the reminder. Approximation of simple functions by Taylor's polynomials.

Antiderivative, indefinite integral. Existence of an antiderivative and an indefinite integral on an interval. Basic indefinite integrals. Integration by parts.

10) Integration by substitution. Application to simple examples.

Rational functions – introduction. Integration of a rational function with polynomial of at most 3rd degree in the denominator.

11) Integration of functions of the type $\sin^m x \cdot \cos^n x$.

Integration of irrational functions of the type $R(x,\ \sqrt[n]{(ax+b)/(cx+d)})$.

Definite (Riemann's) integral, its geometrical and physical interpretation, basic properties.
Newton–Leibniz formula. Integration by parts in definite (Riemann's) integral.

12) Integration by substitution in definite integral. Geometrical applications of definite integral: area of a surface, volume of a rotationally symmetric body, length of a curve.

Riemann's integral as a function of the upper limit. Improper Riemann's integral.