

Mathematics II – Examples

II. Differential Calculus of Several Variables

II.1. Domain of definition of $z = f(x, y)$

- Determine and draw domains of definition for the following functions:

Example 42: $z = \frac{\ln(x^2y)}{\sqrt{y-x}}$

Example 43: $z = \arcsin \frac{y-1}{x}$

Example 44: $z = \frac{\sqrt{x-y^2}}{\ln(1-x^2-y^2)}$

Example 45: $z = \sqrt{\ln \frac{16}{x^2+y^2}}$

Example 46: $z = 3 - 7 \ln(x + \ln y)$

Example 47: $z = \sqrt{2x+y-4} + \sqrt{16-x^2-y^2}$

Example 48: $z = \frac{3}{\sqrt{1-|x|-|y|}}$

Example 49: $z = \sqrt{xy-4}$

- Determine and draw domain of definition, write the set $\text{gr}(f)$ (graph of the function) and sketch the surface in \mathbb{E}_3 , which represents the graph of the given function:

Example 50: $f(x, y) = 4 - \sqrt{x^2 + y^2}$

Example 51: $f(x, y) = \sqrt{x - y^2} + 2$

Example 52: $f(x, y) = \sqrt{x^2 - 9y^2 - 36}$

II.2. Limits and continuity

- Compute the following limits:

Example 53: $\lim_{[x,y] \rightarrow [0,0]} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$

Example 54*: $\lim_{[x,y] \rightarrow [1,1]} \frac{x^3 - y^3}{x^4 - y^4}$, $[x, y] \in M$, where $M = \{[x, y] \in \mathbb{E}_2; x - y \neq 0\}$

Example 55: $\lim_{[x,y] \rightarrow [0,0]} \frac{e^{2(x^2+y^2)} - 1}{x^2 + y^2}$

Example 56: $\lim_{[x,y] \rightarrow [0,0]} \frac{\text{tg}(x^2 + y^4)}{3(x^2 + y^4)}$

Example 57: Determine continuity of the function $f(x, y) = \begin{cases} \frac{x^2y^2}{\sqrt{x^2y^2+1}-1}, & [x, y] \neq [0, 0] \\ 2, & [x, y] = [0, 0] \end{cases}$
in the point $[0, 0]$.

- Find domains of definition for the following functions:

Example 58: $f(x, y) = \frac{x^2 + x - 12}{x^2 + y^4 + 1}$

Example 59: $f(x, y, z) = e^{z^2+x} \cdot \sin(x + y)$

Example 60: $f(x, y) = \frac{1}{x^2 - 2y}$

Example 61: $f(x, y) = \frac{x^4 - y^4}{x^2 + y^2}$

Example 62: $f(x, y, z) = \frac{1}{\ln \sqrt{(x^2 + y^2 + z^2)}}$

Example 63: $f(x, y, z) = \frac{\sin(x^2 + y^2 + z^2)^2}{x^2 + y^2 + z^2}$

Example 64: $f(x, y, z) = \frac{1}{|xy| + |z|}$

Example 65: $f(x, y, z) = \frac{y + 4}{x^2y - xy + 4x^2 - 4x}$