

Mathematics II – Examples

I. Riemann integral

I.1. Existence of Riemann integral

Determine whether there exist the following integrals:

Example 1: $\int_0^1 \frac{x+3}{x^2+1} dx$

Example 4: $\int_0^1 \frac{x+3}{x^2+1} dx$

Example 2: $\int_1^{10} \frac{x^2+3}{x^3-3x^2-4x} dx$

Example 5: $\int_0^2 xe^{x^2} dx$

Example 3: $\int_0^1 \frac{e^{2x}-1}{x} dx$

Example 6: $\int_0^{\pi/2} \frac{1}{1-2\cos x} dx$

Compute the following integrals using Newton-Leibnitz formula:

Example 7: $\int_0^{\pi/4} \frac{1+\sin^2 x}{\cos^2 x} dx$

Example 12: $\int_{-\pi/2}^{\pi/2} x^2 \sin x dx$

Example 8: $\int_3^8 \sqrt{1+x} dx$

Example 13: $\int_0^{\pi} \left| \frac{1}{2} - \cos x \right| dx$

Example 9: $\int_0^2 \frac{x-3}{x^2+4} dx$

Example 14*: $\int_{-4}^4 \frac{1}{\sqrt{x^2+9}} dx$

Example 10: $\int_2^5 \frac{5x+1}{x^2+x-2} dx$

Example 15: $\int_0^{\pi} \frac{\sin x}{2+\cos x} dx$

Example 11: $\int_{-\pi/2}^{\pi/2} \cos^2 \frac{x}{2} dx$

Example 16: $\int_0^1 \frac{e^x}{e^x-e} dx$

Example 17: $\int_{-2}^{-1} \frac{2}{x^2-x} dx$

Compute the following integrals using integration by parts:

Example 18: $\int_{-\pi/2}^{\pi/2} |x| \cos x \, dx$

Example 23*: $\int_{-\pi/2}^{\pi/2} \sin^8 x \, dx$

Example 19: $\int_0^{e-1} \ln(x+1) \, dx$

Example 24*: $\int_0^{\pi} \cos^6 x \, dx$

Example 20: $\int_0^{\pi/2} e^{2x} \sin x \, dx$

Example 25*: $\int_{-\pi/2}^{\pi/2} \sin^9 x \, dx$

Example 21*: $I_n = \int_0^{\pi/2} \sin^n x \, dx$

Example 26: $\int_0^{\sqrt{3}} x \cdot \arctg x \, dx$

Example 22*: $\int_0^{\pi/2} \sin^7 x \, dx$

Example 27: $\int_{1/e}^e |\ln x| \, dx$

Example 28: $\int_0^1 y \cdot \ln(x+y) \, dx, (y > 0)$

Compute the following integrals by substitution:

Example 29: $\int_1^{e^3} \frac{1}{x\sqrt{1+\ln x}} \, dx$

Example 35: $\int_{-2}^{-1} \frac{dx}{x^2 + 4x + 5}$

Example 30: $\int_{1/\pi}^{2/\pi} \frac{1}{x^2} \cdot \sin \frac{1}{x} \, dx$

Example 36: $\int_0^{1/2} \frac{\arcsin x}{\sqrt{1-x^2}} \, dx$

Example 31: $\int_0^1 \frac{x}{1+x^4} \, dx$

Example 37: $\int_0^{\pi/4} \sin^5 x \cdot \cos x \, dx$

Example 32: $\int_0^2 \sqrt{4-x^2} \, dx$

Example 38: $\int_0^{\pi/3} \sin^3 x \, dx$

Example 33: $\int_1^2 \frac{x}{(x^2+4)^2} \, dx$

Example 39: $\int_0^{1/2} r\sqrt{1-4r^2} \, dr$

Example 34: $\int_0^{\pi/2} \frac{\cos x}{\sqrt{\sin^2 x + 3}} \, dx$

Example 40: $\int_0^4 \frac{1}{\sqrt{2x+1}+1} \, dx$

Example 41: $\int_0^{\pi/2} \frac{dx}{2+\cos x}$