Mathematics II – Examples

II.10. Extrema

Necessary condition for existence of local extremum: Let the functions $f(x, y)$ is differentiable in a point A. If f has an local extremum in the point A, then $\mathrm{grad} f(A) = \vec{0}$.

Let us denote

$$\triangle_1(A) = \frac{\partial^2 f}{\partial x^2}(A), \quad \triangle_2(A) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2}(A), & \frac{\partial^2 f}{\partial x \partial y}(A) \\ \frac{\partial^2 f}{\partial y \partial x}(A), & \frac{\partial^2 f}{\partial y^2}(A) \end{vmatrix}$$

(The determinant $\triangle_2$ is called Hessian.)

Sufficient condition for existence of local extremum: Let the functions $f(x, y)$ has continuous partial derivatives of second order in a point A. Let the necessary condition $\mathrm{grad} f(A) = \vec{0}$ holds. Then

If $\triangle_1(A) > 0$ and $\triangle_2(A) > 0$, then the function f has sharp local minimum in point A.
If $\triangle_1(A) > 0$ and $\triangle_2(A) < 0$, then the function f has sharp local maximum in point A.
If $\triangle_1(A) < 0$, then the function f has no extremum in point A.