# Průběh funkce

ALFA, částečně i BETA

**Př.** $f(x)=\frac{\ln x}{x}$

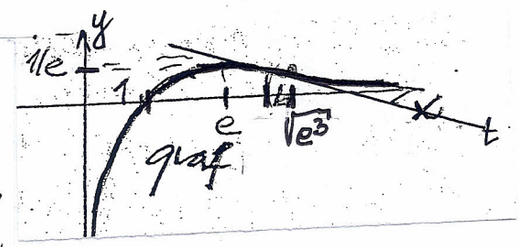
(I.) $D(f)=(0,+\infty)$, f spojitá v $D(f)$

$\lim\limits_{x\to+\infty}=\frac{+\infty}{+\infty}=\frac{\infty}{\infty}\overset{l'H}{=}\lim\frac{\frac{1}{x}}{1}=\frac{0}{1}=0$

$x\to0_+=\text{"}\frac{-\infty}{0_+}\text{"}=-\infty\to$ svislá as. $x=0$

průs. s osou x: $f(x)=0\Leftrightarrow\ln x=0$, $x=1$

GRAF.




(II.) $f'(x)=\frac{1-\ln x}{x^2}$, $D(f')=(0;\infty)$

$f'>0\Leftrightarrow1-\ln x>0$, $1>\ln x$

$x<e^1$, f rost. v $(0;e\rangle$

$f'<0\Leftrightarrow\ldots$, f kles. v $\langle e;+\infty)$

lok. Max $f(e)=1/e$ (absolutní na $D(f)$)

(III.) $f''(x)=\frac{2x\cdot\ln x-3x}{x^4}$, $D(f'')=(0,\infty)$

$f''>0\Leftrightarrow2\ln x-3>0$, $x>e^{3/2}$

$x>\sqrt{e^3}$, f konvex. v $\langle\sqrt{e^3},\infty)$

$f''<0\Leftrightarrow\ldots x<\sqrt{e^3}$, konkáv. v $(0;\sqrt{e^3}\rangle$

$x=\sqrt{e^3}$ Infl. bod $x\doteq4{,}5$

(IV.) $\lim\limits_{x\to+\infty}f(x)=0\Rightarrow$ y=0 je asympt. pro $x\to+\infty$

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**Př.** (I) $f(x)=x^2e^x$; $f'=e^x(2x+x^2)$, $D(f)=D(f')=R$

(II) přímo nerovnice $e^x\cdot x(2+x)\gtrless0$ — f je rost. na $(-\infty;-2\rangle$, f je kles. na $\langle-2;0\rangle$, f je rost. na $\langle0;\infty)$

NEBO nulové body derivace

lok. max v $x=-2$; $f(-2)=4/e^2$

lok. min v $x=0$, $f(0)=0$

(III) $f''=e^x(x^2+4x+2)=0$

$x_{1,2}=-2\pm\sqrt2\doteq\begin{cases}-0{,}6\\-3{,}4\end{cases}$

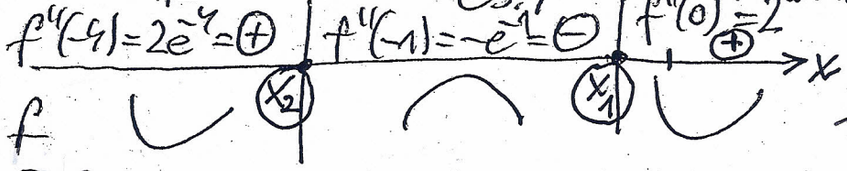
f konvex. v $(-\infty,x_2\rangle$

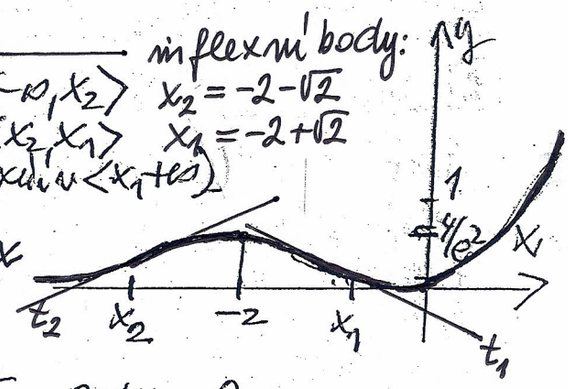
f konkáv. $\langle x_2,x_1\rangle$

f konvex. v $\langle x_1,+\infty)$

inflexní body: $x_2=-2-\sqrt2$, $x_1=-2+\sqrt2$

$f''(-4)=2e^{-4}=\oplus$ | $f''(-1)=-e^{-1}=\ominus$ | $f''(0)=2$ $\oplus$




(IV) $\lim\limits_{x\to\infty}=\infty\cdot\infty=\infty$; $\lim\limits_{x\to-\infty}=\text{"}\infty\cdot0\text{"}=\lim\frac{x^2}{e^{-x}}=2\,l'H=0\Rightarrow$

přímka y=0 je asymptota pro $x\to-\infty$