Czech Technical University in Prague
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Mathematics II, level Alpha – Requests for exams in academic year 2015/16

1. Riemann integral of a function of one variable.

2. Interior and boundary points of a subset of $\mathbb{E}_n$. Border and closure of a set in $\mathbb{E}_n$. Open, closed and bounded sets; domain.

3. Functions of several variables: domain, continuity, graph, first order partial derivatives, geometric interpretation.

4. Sets in $\mathbb{E}_2$ with a 2D curve as a border (straight line, conic section, graphs of functions of one variable). Sets in $\mathbb{E}_3$ with 3D surfaces as a border (plane, quadratic surfaces, graphs of functions of two variables).

5. Total differential, sufficient conditions. Tangent plane and normal line to the graph of a function $z = f(x, y)$ or to the surface given by the equation $F(x, y, z) = 0$. Approximative value of a function using differential or tangent plane, respectively. Gradient of a function, geometrical and physical meaning.
   Directional derivative, its geometrical meaning.

6. Partial derivatives of higher order. Differential operators. Divergence of a vector field. Rotation of a vector field. Physical interpretation.

7. Function of one variable $y = f(x)$ defined by implicit way by the equation $F(x, y) = 0$ (existence theorem, continuity theorem and evaluation of derivatives of first and second order). Monotony, local extremes, convexity (concavity), Taylor's polynomial of the second order. Tangent line to the implicit function of one variable. Approximation of a function value.

8. Function of two variables $z = f(x, y)$ defined by implicit way using the equation $F(x, y, z) = 0$ (existence theorem, continuity theorem, evaluation of partial derivatives). Tangent plane to the graph of an implicit function of two variables. Approximation of a function value. Gradient, differential, directional derivative of a implicit function of two variables.

9. Local extremes of a function $z = f(x, y)$. Necessary and sufficient condition. Global extremes.

10. Double integral. Fubini's theorem. Geometrical and physical applications. Area of a planar shape. Mass, center of gravity, static moment, moment of inertia of a flat plate.

11. Triple integral. Fubini's theorem. Geometrical and physical applications. Volume of a 3D-area. Mass, center of gravity, static moment, moment of inertia of a body.

12. Basic properties of double and triple integral. Computation of double and triple integrals using transformations into polar, cylindrical and/or spherical coordinates. Use of generalized versions of these coordinate systems.(polar, cylindrical).

13. Simple, (piecewise) smooth curve in $\mathbb{E}_2$ or in $\mathbb{E}_3$, its parametrization. Closed curve. Line integral of a scalar function, basic properties. Application of line integral in physics (mechanical characteristics - see 10). Length of a curve.

14. Line integral of a vector function, physical meaning (work done along a given curve by a given force). Circulation of 2D-vector field around a closed curve in $\mathbb{E}_2$. Green theorem.

15. Conservative vector field in $\mathbb{E}_2$ and in $\mathbb{E}_3$. Necessary and sufficient conditions for existence of potential function. Independence of line integral of path. Circulation of a vector field, computation of a potential function in $\mathbb{E}_2$ and/or in $\mathbb{E}_3$ (simple examples only).

16. Simple (piecewise) smooth surface in $\mathbb{E}_3$, parametrization. Quadric surfaces. Surface as a graph of a function of two variables. Surface integral of a scalar function, basic properties. Physical applications (mechanical characteristics - see 10). Surface's area.

17. Surface integral of a vector function, basic properties. Physical applications (flow of a vector field through a given surface). Gauss-Ostrogradski theorem.