Mathematics 2 - 2. test

var. 1

full name: .....

1. Write the Jacobian for transformation $(x, y, z) \mapsto (r, \theta, \varphi)$ given:

   $x = r\cos\varphi\cos\theta$
   $y = r\sin\varphi\cos\theta$
   $z = r\sin\theta.$ $\qquad |J| = \ldots\ldots\ldots\ldots\ldots\ldots$

2. Convert the given integral to cylindrical coordinates.
   DON'T COMPUTE further!

   $$\iiint\limits_{T} \frac{\sqrt{1-x^2-y^2}}{xz}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \iiint\limits_{T*} \ldots\ldots\ldots\ldots\ldots\ldots$$

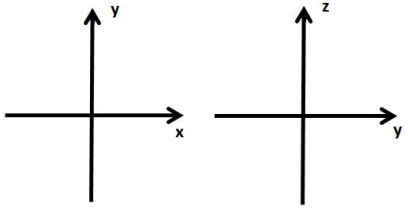
3. Convert the given integral to spherical coordinates.
   DON'T COMPUTE further!

   $$\iiint\limits_{D} \frac{\sqrt{(x^2+y^2)}}{(x^2+y^2+z^2)}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \iiint\limits_{D^*} \ldots\ldots\ldots\ldots\ldots\ldots$$

4. Compute:

   $$\int_0^2 \int_0^{\sqrt{1-x}} xy\,\mathrm{d}y\,\mathrm{d}x = \ldots\ldots\ldots\ldots\ldots\ldots$$

5. Sketch a body given by: $x^2 + y^2 \le 1 \ \wedge$
   $\wedge\ 0 \le z \le 2 - x^2 - y^2$ in a given given cuts.

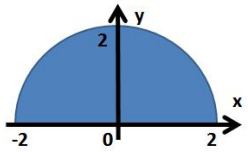


Mathematics 2 - 2. test

var. 2

full name: .....

1. Compute y coordinate of a mass center of a half-disc with $\rho = 2$.
   For mass use $m = \frac{\pi R^2 \rho}{2}$.



   $y_{CM} = \ldots\ldots\ldots\ldots\ldots$

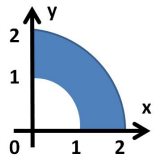
2. Name the body which is described by following inequalities. Write the coordinates of a center and which part of the body is it.

   $0 \le z \le \sqrt{1-x^2-y^2}$ $\qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

3. Compute:

   $$\int_0^{\sqrt{2}} \int_0^{\frac{\pi}{2}} \int_0^{2} r\cos\varphi\,\mathrm{d}z\,\mathrm{d}\varphi\,\mathrm{d}r = \ldots\ldots\ldots\ldots\ldots\ldots$$

4. Describe the displayed region by inequalities.



   $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

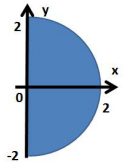
5. Write at least two physical meanings of following integral (for each case determine ρ).

   $$\int_0^1 \int_0^1 x^2 y\,\mathrm{d}x\,\mathrm{d}y$$

   1) $\rho = \ldots\ldots \mapsto \ldots\ldots\ldots\ldots\ldots\ldots$
   2) $\rho = \ldots\ldots \mapsto \ldots\ldots\ldots\ldots\ldots\ldots$

var. 1

6. Compute x coordinate of a mass center of a half-disc with $\rho = 1$. For mass use $m = \frac{\pi R^2 \rho}{2}$.



$x_{CM} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

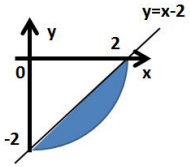
7. Name the body which is described by following inequalities. Write the coordinates of a top and which part of the body is it.

$0 \le z \le 2 + \sqrt{x^2 + y^2}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

8. Compute

$$\int_0^{\sqrt{2}} \int_0^{\frac{\pi}{2}} \int_0^{2\pi} r\, r^2 \cos\theta \,\mathrm{d}\varphi \,\mathrm{d}\theta \,\mathrm{d}r = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

9. Describe the displayed body by inequalities.



$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

10. Write at least two physical meanings of following integral (for each case determine ρ).

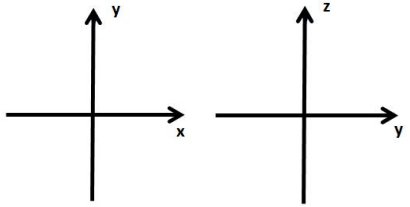
$$\int_0^1 \int_0^1 xy \,\mathrm{d}x \,\mathrm{d}y$$

1) $\rho = \ldots\ldots \mapsto \ldots\ldots\ldots\ldots\ldots\ldots$

2) $\rho = \ldots\ldots \mapsto \ldots\ldots\ldots\ldots\ldots\ldots$

var. 2

6. Sketch a body given by: $x^2+y^2 \le 1 \ \wedge \ 0 \le z \le 1+x^2+y^2$ in a given cuts.



7. Compute:

$$\int_0^2 \int_0^{\sqrt{1+y}} xy \,\mathrm{d}x \,\mathrm{d}y = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

8. Convert the given integral to cylindrical coordinates. DON'T COMPUTE further!

$$\iiint_D \frac{\sqrt{(x^2+y^2)^3 - 10}}{\sqrt{(x^2+y^2+z^2)}} \,\mathrm{d}x \,\mathrm{d}y \,\mathrm{d}z = \iiint_{D^*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

9. Convert the given integral to polar coordinates. DON'T COMPUTE further!

$$\iint_D \frac{5xy}{\sqrt{x^2+y^2}} \,\mathrm{d}y \,\mathrm{d}x = \iint_{D^*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

10. Write the Jacobian for transformation $(x, y) \mapsto (r, \varphi)$ given:

$x = 1 + r\cos\varphi$

$y = -1 + r\sin\varphi$ $\qquad |J| = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$