Mathematics I A – Exam 2

1. a) Compute the determinant of the matrix of the system with parameter $a \in \mathbb{R}$.

$$\begin{aligned} ax + ay + 2z &= -2 \\ x + ay &= 4 \\ y - z &= 2. \end{aligned}$$

b) Explain the Cramer's rule for the solution of a system of linear equations.
Find the values of parameter a, for which it's *not possible* to use Cramer's rule.

c) Using Cramer's rule compute the unknown x, when $a = -3$.

2. a) Define the notion of an inverse matrix to a given matrix $\boldsymbol{A}$. Write down, how the existence or non-existence of the inverse matrix is related to the existence and number of solutions to the system $\boldsymbol{A}\vec{x} = \vec{0}$.

b) Does there exists the inverse matrix to the given matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 5 \end{pmatrix}$?
Justify your answer!
If the inverse matrix exists, find it and verify the result (from definition).

c) Find the matrix $\boldsymbol{X}$,
which is the solution of the equation $\boldsymbol{A} \cdot \boldsymbol{X} = \boldsymbol{B}$, where $\boldsymbol{B} = \begin{pmatrix} 3 & -2 \\ -1 & 0 \\ 0 & 5 \end{pmatrix}$.

3. Given function $f(x) = \sqrt{2x+6} - x^2$.

a) Compute the derivative $f'(x)$ and find the domains $D(f)$ a $D(f')$.

b) Find the equation of a tangent to the graph of the function f at point $[x_0; f(x_0)]$ where $x_0 = -1$.

c) Compute the $f''(x)$ and write down the Taylor's polynomial $T_2(x)$ of second order centered at $x_0 = -1$ for the given function f.

d) Find the estimate of the maximum error caused by replacing the values of f by the above Taylor's polynomial $T_2(x)$ at interval $\langle -1; 0 \rangle$.

4. Given function $f(x) = \dfrac{1-x}{x^2+3}$.

a) Find the domain $D(f)$ and the intersections of the graph of f with coordinate axes.

b) Find the intervals of monotonicity and local extrema of the function f (i.e. their type, position, values).

c) Find the limits of $f(x)$ at the boundary points of the domain $D(f)$ and sketch the graph.

5. Find the following integrals. Don't forget the intervals of their existence.

a) $\displaystyle\int (x^2 - 1) \ln x \, \mathrm{d}x$ b) $\displaystyle\int \left(4x - \cos^3(2x)\right) \mathrm{d}x$

6. a) Sketch the region bounded by the curves $y = \sqrt{x+4}$, $x = 0$ and $y = 0$ and find its surface area.

c) Compute the volume of the body that arises by rotation of this region around the x axis.

d) Compute the volume of the body that arises by rotation of this region around the y axis.

Mathematics I B – Exam 2

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1. a) Compute the determinant of the matrix of the system:

$$\begin{aligned} x + \ y - 2z &= 1 \\ 3x + 2y + 2z &= -2 \\ 2x + \ y - \ z &= 2. \end{aligned}$$

b) Explain the use of the Cramer's rule for the solution of the systems of linear equations with square matrix $\boldsymbol{A}$. Find out if it's possible to use the Cramer's rule for the solution of the above given system.

c) Using the Cramer's rule compute the value of the unknown y.

2. Given matrices $\boldsymbol{A} = \begin{pmatrix} 2 & -1 \\ 4 & -1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 2 & 0 \\ 6 & -2 \end{pmatrix}$.

a) Justify the existence of the inverse matrix $\boldsymbol{A}^{-1}$. Find the inverse matrix $\boldsymbol{A}^{-1}$.

b) Express the unknown matrix $\boldsymbol{X}$ from the matrix equation $\boldsymbol{A} \cdot \boldsymbol{X} = \boldsymbol{B}$.
Compute the matrix $\boldsymbol{X}$.

c) Find the matrix $\boldsymbol{Y} = \boldsymbol{B} \cdot \boldsymbol{A}$.

3. Given function $f(x) = x^2 + 4x + 1$.

a) Find the equation of a tangent to the graph of the function f at point $[x_0; f(x_0)]$ where $x_0 = 1$.

b) Using this tangent compute the approximate value $f(1.5)$.
What is the error (replacing the exact value by the approximate one) in this case?

c) Write down the Taylor's polynomial of given function centered at
i) $x_0 = 0$, ii) $x_0 = 1$

4. Given function $f(x) = \dfrac{1}{9 - x^2}$.

a) Find the domain $D(f)$. Is the function odd or even? Justify you answer.

b) Compute the derivative $f'(x)$. Find the points where the $f'(x)$ is equal zero.

c) Find the intervals of monotonicity and local extrema at the interval $(-3; 3)$.

d) Find the limits of f for $x \to -3_+$ (from right), for $x \to 3_-$ (from left).
Find the minimum and maximum of the given function f at $(-3; 3)$.
Sketch the graph of the function at this interval.

5. Find the integrals a) $\int 3x^2 \ln x \, \mathrm{d}x$, b) $\int \Big(\cos(3 - 2x) + x\Big) \, \mathrm{d}x$.
Write down the intervals of existence of these integrals.

6. a) Compute the integral $\int_0^4 \sqrt{x} \, \mathrm{d}x$.

b) Sketch the domain bounded by the x axis and by the graphs of the functions $y = \sqrt{x}$, $y = 6 - x$. Compute the surface area of this domain.

c) Find the volume of the body that arises by rotation of the above domain around the x axis.