

## Mathematics I – Exam 2

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1. a) Compute the determinant of the matrix of the system with parameter  $a \in \mathbb{R}$ .

$$\begin{aligned} ax + ay + 2z &= -2 \\ x + ay &= 4 \\ y - z &= 2. \end{aligned}$$

- b) Explain the Cramer's rule for the solution of a system of linear equations.  
Find the values of parameter  $a$ , for which it's *not possible* to use Cramer's rule.
- c) Using Cramer's rule compute the unknown  $x$ , when  $a = -3$ .

2. a) Define the notion of an inverse matrix to a given matrix  $A$ . Write down, how the existence or non-existence of the inverse matrix is related to the existence and number of solutions to the system  $A\vec{x} = \vec{0}$ .

- b) Does there exist the inverse matrix to the given matrix  $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 5 \end{pmatrix}$ ?  
Justify your answer!

If the inverse matrix exists, find it and verify the result (from definition).

- c) Find the matrix  $X$ ,  
which is the solution of the equation  $A \cdot X = B$ , where  $B = \begin{pmatrix} 3 & -2 \\ -1 & 0 \\ 0 & 5 \end{pmatrix}$ .

3. Given function  $f(x) = \sqrt{2x+6} - x^2$ .

- a) Compute the derivative  $f'(x)$  and find the domains  $D(f)$  and  $D(f')$ .
- b) Find the equation of a tangent to the graph of the function  $f$  at point  $[x_0; f(x_0)]$  where  $x_0 = -1$ .
- c) Compute the  $f''(x)$  and write down the Taylor's polynomial  $T_2(x)$  of second order centered at  $x_0 = -1$  for the given function  $f$ .
- d) Find the estimate of the maximum error caused by replacing the values of  $f$  by the above Taylor's polynomial  $T_2(x)$  at interval  $\langle -1; 0 \rangle$ .

4. Given function  $f(x) = \frac{1-x}{x^2+3}$ .

- a) Find the domain  $D(f)$  and the intersections of the graph of  $f$  with coordinate axes.
- b) Find the intervals of monotonicity and local extrema of the function  $f$  (i.e. their type, position, values).
- c) Find the limits of  $f(x)$  at the boundary points of the domain  $D(f)$  and sketch the graph.

5. Find the following integrals. Don't forget the intervals of their existence.

a)  $\int (x^2 - 1) \ln x \, dx$       b)  $\int (4x - \cos^3(2x)) \, dx$

6. a) Sketch the region bounded by the curves  $y = \sqrt{x+4}$ ,  $x = 0$  and  $y = 0$  and find its surface area.

- c) Compute the volume of the body that arises by rotation of this region around the  $x$  axis.
- d) Compute the volume of the body that arises by rotation of this region around the  $y$  axis.