Mathematics I – Exam 4

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1. a) Explain the Cramer's rule (don't forget the assumptions).

b) Find all values of the parameter $\lambda \in \mathbb{R}$, for which it's possible to use the Cramer's rule to solve the given system of linear algebraic equations for unknowns $x,\ y,\ z$:

$$\begin{aligned} 3x + y &= \lambda(1-z) + 1 \\ x + y &= \lambda \\ 2(x+y-z) &= \lambda(1-y). \end{aligned}$$

c) Use the Cramer's rule to compute the unknown y from the system given in b), for the value $\lambda = -3$.

2. a) Define the notion of the *inverse matrix*. Write some necessary and sufficient condition for the existence of an inverse matrix to the given matrix $\boldsymbol{A}$.

b) Verify if there exists an inverse matrix to the matrix $\boldsymbol{A} = \begin{pmatrix} 2, & 1, & 0 \\ 1, & 0, & -1 \\ 0, & 1, & 3 \end{pmatrix}$.

If it exists, compute it and verify the result (from definition).

c) Find the inverse matrix to the matrix $\boldsymbol{B} = \boldsymbol{A}^2$.

3. Given function $f(x) = x(5x-1)^{-1/2}$.

a) Compute the derivative $f'(x)$. Find the domains $D(f),\ D(f')$.

b) Write the equations of the tangent and normal lines to the graph of the given function at the point $[x_0, f(x_0)]$ for $x_0 = 2$.

c) Justify the existence of absolute extrema of the given function on the interval $\langle 1/4, 3\rangle$. Find those extrema, i.e. find their position, type and (approximate) value.

4. Given function $f(x) = \dfrac{x+1}{x^2-4}$.

a) Find the domain $D(f)$ of the function f, find the intersections of the graph of f with the x and y axes, find the limits of f in the boundary points of $D(f)$.

b) Determine the intervals of monotonicity and local extrema of f (i.e. find their position, type, value)

c) Find all asymptotes of the graph of the function f. Sketch the graph.

5. Find the following integrals and intervals of their existence.

a) $\displaystyle\int \ln(x+1)\,\mathrm{d}x$ b) $\displaystyle\int \frac{(3x+4)}{(x+1)(x+2)^2}\,\mathrm{d}x$

6. Given function $f(x) = x\,\sin(x^2+1)$.

a) Compute the integral $\int f(x)\mathrm{d}x$. Verify the result (from the definition).

b) Compute the (integral) mean value of the function f on the interval $\langle 0; \sqrt{2\pi}\rangle$.