Some notes on Mathematics I

December 7, 2025

Notation & terminology

Notation and terminology will closely follow the official textbook and sample exercises. No other notation, terminology and solution procedures will be accepted.

Neustupa, J.: Mathematics I, CTU Publishing House, Prague, 1996

Neustupa, J.: Mathematics I, updated electronic version (available on our web)

Collection of examples from Mathematics I (written in Czech) by authors E. Brožíková, M. Kittlerová (available on our web)

The exam from Mathematics should also verify your ability to write properly the complete solution procedure, including the knowledge and proper use of the standard mathematical notation. Any missing or inappropriate description of the solution procedure or wrong use of mathematical notation will lead to significant reduction of points you will obtain for the exercise.

Number sets & Spaces

The standard number sets are denoted by capital letters $\mathbb{R}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{C}$ using a double stroke fonts to avoid confusion with some other objects. It means that the double stroke $\mathbb{R}$ denotes something different than standard capital R. The double stroke letters should also be preferred everywhere in your handwritten solutions.

$\mathbb{R}$	set of real numbers	$\mathbb{R}^n$	n-dimensional real space
$\mathbb{C}$	set of complex numbers	$\mathbb{C}^n$	n-dimensional complex space
$\mathbb{N}$	set of natural numbers	$\mathbb{E}_n$	n-dimensional Euclidian space
$\mathbb{Q}$	set of rational numbers	$\mathbb{R}^{m\times n}$	space of real $m \times n$ matrices

The space of continuous functions on a set G is denoted $C(G)$. If the derivatives are also continuous up to order n, the space is denoted $C^n(G)$. The space of Riemann integrable functions is denoted $R(G)$.

Brackets

$$[point] \qquad A = [1;2]$$

$$(vector) \qquad \vec{a} = (1;2)$$

$$\{set\} \qquad D = \{[x;y] \in \mathbb{E}_2 : x > y\}$$

The round brackets ( ) are used in most cases in different sizes as a standard. The curly brackets { } are only used for sets. The square brackets [ ] are either used for points, or for antiderivatives in definite integral accompanied by corresponding bounds $[F(x)]_a^b$.

Intervals

$$x \in (0;1) \qquad \Longleftrightarrow \qquad 0 < x < 1$$

$$x \in \langle 0;1) \qquad \Longleftrightarrow \qquad 0 \leqq x < 1$$

$$x \in (0;1\rangle \qquad \Longleftrightarrow \qquad 0 < x \leqq 1$$

$$x \in \langle 0;1\rangle \qquad \Longleftrightarrow \qquad 0 \leqq x \leqq 1$$

Standard functions

$\sin x$	$\arcsin x$	a^x	$\log_a x$
$\cos x$	$\arccos x$	10^x	$\log x = \log_{10} x$
$\tan x$	$\arctan x$	e^x	$\ln x = \log_e x$
$\operatorname{cotan} x$	$\operatorname{arccotan} x$		

The powers of functions are denoted by an upper index (exponent), like in:

$$\ln^2 x = (\ln x)^2 \qquad \text{and} \qquad \ln^{-2} x = \frac{1}{(\ln x)^2}$$

Especialy note that for example

$$\tan^{-1} x = \frac{1}{\tan x} = \frac{\cos x}{\sin x} = \operatorname{cotan} x \neq \arctan x$$

1. Differential calculus

1.1. Functions

$f(x)$	*scalar* function of *scalar* variable	$x \in \mathbb{R} \mapsto f(x) \in \mathbb{R}$
$f(\vec{x})$	*scalar* function of *vector* variable	$\vec{x} \in \mathbb{R}^n \mapsto f(\vec{x}) \in \mathbb{R}$
$\vec{f}(x)$	*vector* function of *scalar* variable	$x \in \mathbb{R} \mapsto \vec{f}(x) \in \mathbb{R}^n$
$\vec{f}(\vec{x})$	*vector* function of *vector* variable	$\vec{x} \in \mathbb{R}^n \mapsto \vec{f}(\vec{x}) \in \mathbb{R}^m$

The scalar function of vector variable is called *function of multiple variables*

$$f(\vec{x}) = f(x_1; \ldots; x_n) \qquad \text{where} \qquad \vec{x} = (x_1; \ldots; x_n) \qquad \vec{x} \in \mathbb{R}^n \mapsto f(\vec{x}) \in \mathbb{R}$$

The function f is defined on a set $D(f) \subseteq \mathbb{R}^n$ called the domain of f.
The domain must always be written as a set.

For example the function $g(x, y, z) = \ln(x + y)$ is defined on

$$D(g) = \{[x; y; z] \in \mathbb{E}_3 : x + y > 0\}$$

1.2. Local extrema

- By determining the local extrema of a function you should always find and specify the *position*, *type* and *value* for each extremum.
- The answer should be written and clearly formulated. For example:
  Function f has local minimum at point $x = 1$ where $f(1) = 3$.
  Function f has local minimum 3 at $x = 1$.
  Similar answers should also be formulated for global (absolute) extrema.

1.3. Global extrema

- The procedure of finding global extrema is significantly different from finding local extrema. For global extrema the general principle is:
  Make a list of "suspicious" points (their positions) and then compute and compare the function values in those points. Finally, select the biggest and smallest from these values.
- "Suspicious" points are the *critical points from the interior* of the given set and *all boundary points*.
- For the internal critical points (where $f' = 0$ or $\nexists f'$), we do not study the type of local extrema at that point. We just put that point on the list of "suspicious" points for function value comparison. So, there is no need to compute second derivatives of f (for finding global extrema).

2. Integral calculus

- When solving an integral write complete solution procedure in a ”single line”, starting from the original integral, up to final result, without any interruption.
- Any auxiliary calculations (transformations, substitutions, by-part decomposition, etc.) should be written ”inline” in this solution between vertical lines.
- For indefinite integral do not forget to always write down the interval of the existence of the integral.
- For definite integral solved using substitution, recalculate the bounds for the transformed integral.

2.1. Indefinite integral

Compute $$\int (x-1)\ln x \, dx$$

$$\int (x-1)\ln x \, dx = \left| \begin{array}{ll} u = \ln x, & v' = x - 1 \\ u' = \frac{1}{x}, & v = \frac{x^2}{2} - x \end{array} \right| = \left(\frac{x^2}{2} - x\right) \ln x - \int \frac{1}{x} \cdot \left(\frac{x^2}{2} - x\right) dx =$$

$$= \left(\frac{x^2}{2} - x\right) \ln x - \int \left(\frac{x}{2} - 1\right) dx = \left(\frac{x^2}{2} - x\right) \ln x - \frac{x^2}{4} + x + C \,,$$

$$x \in (0,\ \infty).$$

2.2. Definite integral

Compute $\displaystyle\int_0^2 \sqrt{4-x^2}\,dx$

$$\int_0^2 \sqrt{4-x^2}\,dx = \left| \begin{array}{ll} x = 2\sin t & x_1 = 0 \implies t_1 = 0 \\ dx = 2\cos t\,dt & x_2 = 2 \implies t_2 = \pi/2 \end{array} \right| =$$

$$= \int_0^{\pi/2} \sqrt{4-4\sin^2 t}\cdot 2\cos t\;dt = 4\int_0^{\pi/2} \cos^2 t\;dt = 4\int_0^{\pi/2} \frac{1+\cos 2t}{2}\,dt =$$

$$= 2\Big[t + \frac{\sin 2t}{2}\Big]_0^{\pi/2} = 2\cdot\frac{\pi}{2} = \pi.$$

2.3. Applications (mechanical, geometrical)

- When calculating a physical quantity, you should always specify what you are computing (name of the quantity), formula that you are using, integral you are solving, and the final result.

$$quantity = formula = integral = \ldots = result$$

$$m = \int_1^3 \rho(x)\,\mathrm{d}x = \int_1^3 x^2\,\mathrm{d}x = \ldots = \frac{4}{3}$$

- Unless there are physical units mentioned in the question, the result is without any units.