Czech Technical University in Prague
Faculty of Mechanical Engineering
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Mathematics I – Examination Requirements
Academic Year 2026/27

Explanatory notes:

- Knowledge of concepts means that the student is able to define and explain these concepts, state their basic properties (within the scope of the lectures), illustrate them with examples, and work with them in assigned problems (verification, calculation, problem solving).
- To formulate (state) a theorem means to state all assumptions and the complete assertion of the required theorem.
- A description of a method includes both stating the assumptions for applying the method and describing the computational algorithm and the use of the method in solving a specific problem.

In the examination, emphasis will be placed not only on knowledge of the procedures used in solving problems, but also on theoretical knowledge and correct mathematical expression (formulation of assumptions, properties, and results). In written work, the notation and terminology used in the official course textbook (available online) are required.

1. Knowledge of the concepts: vector space, linear combination, linear dependence and independence of vectors, basis, dimension, subspace. Description of the basic operations with vectors (multiplication by a scalar, addition, scalar product and vector product) and their properties. Solving problems with a parameter.

2. Knowledge of the concepts: matrix, rank of a matrix, nonsingular and singular matrix, inverse matrix. Description of the basic operations with matrices and their properties. Solving problems with a parameter. Determinant, its properties and calculation. Solving problems with a parameter.

3. Knowledge of the concept of a system of linear algebraic equations and formulation of the theorem on the existence and number of solutions of a system (Frobenius theorem). Description of methods for solving systems of linear algebraic equations (Gaussian elimination, Cramer's rule, inverse matrix). Solving systems with a parameter.

4. Knowledge of the concepts: eigenvalues and eigenvectors of a square matrix. Description of the method for finding the eigenvalues and eigenvectors of a given matrix of size up to and including 3×3.

5. Knowledge of the concepts: sequence of real numbers, bounded sequence, monotone sequence, subsequence, limit of a sequence. Formulation of theorems on limits of sequences and their application in calculating limits.

6. Knowledge of the concepts: real function of one real variable, domain, range, graph of a function, restriction of a function, composite function, inverse function, bounded function, monotone function, local and global extrema of a function, asymptote of a function.

7. Knowledge of elementary real functions (linear, quadratic, power, nth-root, exponential, logarithmic, trigonometric, and inverse trigonometric functions) and description of their behaviour (graphs).

8. Knowledge of the concepts: limit of a function (finite and infinite), right-hand and left-hand limits. Formulate the basic theorems on limits of functions and describe their application in calculating limits. Knowledge of the concept of continuity of a function. Formulate the basic theorems on continuity of functions: continuity of composite and inverse functions, and properties of continuous functions on a bounded closed interval.

9. Knowledge of the concept of the derivative of a function, including its geometric and physical interpretation, differential of a function, and higher-order derivatives. Knowledge of formulas for derivatives of elementary functions and their derivation. Formulate the basic theorems on differentiation of functions. Describe the method for finding the equation of the tangent to the graph of a function and the use of the differential for approximating a function at a given point.

10. Formulate the mean value theorem and theorems concerning the relationship between the sign of the first derivative and the behaviour of a function at a point and on an interval. Describe the method for calculating the limit of a function using L'Hospital's rule. Describe the method for finding local and global extrema of functions on an interval.

11. Knowledge of the concepts: convex and concave function, inflection point. Formulate the theorems concerning the relationship between the sign of the second derivative and convexity or concavity of a function at a point and on an interval. Describe the method for analysing the behaviour of a specific function.

12. Knowledge of the concepts: Taylor polynomial, Lagrange form of the remainder. Describe the method for approximating a function by a Taylor polynomial, including an estimate of the approximation error.

13. Knowledge of the concepts: antiderivative (including the condition for its existence), indefinite integral. Knowledge of the basic (so-called tabulated) indefinite integrals. Formulate the theorem on the properties of the indefinite integral, the theorem on integration by parts, and the theorem on integration by substitution. Describe methods for calculating indefinite integrals.

14. Describe the method for integrating a rational function whose denominator is a polynomial of degree at most 3: decomposition of a rational function and integration of partial fractions.

15. Describe the method for integrating trigonometric functions of the type

$$f(x) = \sin^m x \, \cos^n x$$

and the method for integrating irrational functions of the type

$$R\left(x,\, \sqrt[n]{\frac{ax+b}{cx+d}}\right).$$

16. Knowledge of the concept of the definite (Riemann) integral and its geometric interpretation (area under the graph of a function). Describe the method for calculating a definite integral using the Newton–Leibniz formula, integration by parts, and the substitution method for the Riemann integral. Knowledge of the concept of the mean value of a function on an interval. Describe applications of the Riemann integral to calculating the area of a plane region, the volume of a solid of revolution, and the length of a curve.

17. Knowledge of the concepts: Riemann integral as a function of the upper limit, improper Riemann integral.