Czech Technical University in Prague
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# Mathematics I – Lecture Schedule for Full-Time Study
# Academic Year 2026/2027

1. **week (21. 9. – 25. 9.) Basic concepts of linear algebra**
   Number sets, the field of real numbers $\mathbb{R}$. Vector space, linear dependence and independence of a set of vectors, dimension, basis, subspace. Special spaces $\mathbb{R}^n$, $\mathbb{E}_n$, and $V(\mathbb{E}_n)$. Operations with vectors in $V(\mathbb{E}_n)$ (addition and subtraction of vectors, multiplication of a vector by a real number, length (norm) of a vector, scalar product, vector product, and scalar triple product).

2. **week (28. 9. – 2. 10.)** (*Monday September 28 is cancelled, will be replaced on January 4.)*
   Matrices of type $m \times n$, transpose matrix, triangular, diagonal, square, identity, and symmetric matrices. Equality of two matrices, operations with matrices (multiplication of a matrix by a real number, sum, difference, and product of two matrices). Rank of a matrix. Determination of the rank of a specific matrix (including matrices with parameters). Elementary row operations on matrices with complex entries (e.g. for calculating eigenvectors).

   Determinant of a square matrix. Properties and calculation of determinants. Nonsingular and singular matrices. Inverse matrix, conditions for its existence, and its calculation. Vector product and scalar triple product of vectors in $\mathbb{E}_3$ using the "determinant" of a $3 \times 3$ matrix. Problems with parameters.

3. **week (5. 10. – 9. 10.)**
   Systems of linear algebraic equations (coordinate and matrix notation). Gaussian elimination. Frobenius theorem. Existence and number of solutions of homogeneous and nonhomogeneous systems, structure of the set of all solutions. Cramer's rule. Solution of systems of linear equations with parameters.

   Eigenvalues and eigenvectors of a square matrix, geometric interpretation. Spectral radius. Characteristic equation of a square matrix. Finding eigenvalues and eigenvectors of a specific matrix for $n = 2$, $n = 3$. Eigenvalues and eigenvectors of powers of a matrix, including the inverse matrix. Relationship between the determinant and trace of a matrix and its eigenvalues (definitions in terms of eigenvalues).

4. **week (12. 10. – 16. 10.) Fundamentals of differential calculus of functions of one real variable**
   Functions of one real variable, domain, range, graph. Restriction of a function. Even, odd, and periodic functions. Composite function. Inverse function. Functions bounded above, bounded below, bounded, increasing, decreasing, non-increasing, non-decreasing, monotone, and strictly monotone. Overview of elementary functions: power functions, polynomials, $n$th roots, linear fractional functions, exponential, logarithmic, and trigonometric functions. Inverse trigonometric functions.

   Extended set of real numbers $\mathbb{R}^*$, operations and ordering in $\mathbb{R}^*$. Definitions of the minimum, maximum, infimum, and supremum of a set. Various types of neighbourhoods of points in $\mathbb{R}^*$. Sequences of real numbers; sequences bounded above, bounded below, bounded, increasing, and decreasing.

   Limit of a sequence. Basic theorems on limits of sequences (arithmetic operations, squeeze theorem,

subsequences), and their use in calculating limits of sequences.

5. **week (19. 10. – 23. 10.)**
Limit of a function (finite and infinite, at finite and infinite points). Right-hand limit, left-hand limit. Basic theorems on limits of functions. Limit of a composite function. Calculation of simple limits.

   Continuity of a function at a point, right and left continuity. Continuity of a function on an interval. Definitions of the minimum, maximum, infimum, and supremum of a function. Theorems on the continuity of the sum, difference, product, and quotient of two functions. Theorem on continuity of a composite function and theorem on continuity of an inverse function. Intermediate value theorem (Darboux theorem) and theorem on the existence of a maximum and minimum of a continuous function on a bounded closed interval. Approximate solution of the equation $f(x) = 0$ by the bisection method (informative).

6. **week (26. 10. – 30. 10.)** (*Wednesday October 28 is cancelled, will be replaced on January 6.)*
Derivative of a function at a point $x_0$, right and left derivatives, infinite derivative. Geometric and physical interpretation of the derivative. Equation of the tangent to the graph of the function $y = f(x)$ at the point $[x_0, f(x_0)]$. Differential of a function at a point, its geometric interpretation, approximate calculation of function values.

   Relationship between the existence of a derivative and continuity of a function at a point and on an interval. Formulas for derivatives of elementary functions (examples of derivations).

   Theorems on the derivative of the sum and difference of functions, a real multiple of a function, and the product and quotient of two functions. Theorems on the derivative of a composite function and an inverse function. Applications to specific examples. Higher-order derivatives. Mean value theorem (Lagrange), graphical illustration. L'Hospital's rule.

7. **week (2. 11. – 6. 11.)**
Relationship between the sign of the first derivative and the behaviour of a function on an interval. Monotonicity of a function. Convex and concave functions on an interval. Inflection point. Relationship between the sign of the second derivative and convexity (concavity) of a function on an interval. Finding inflection points of functions. Approximate solution of the equation $f(x) = 0$ by Newton's method (informative).

   Local extrema of a function and their relationship to the first and second derivatives. Procedure for finding local extrema. Global extrema of a continuous function on an interval. Finding global extrema of functions.

8. **week (9. 11. – 13. 11.)**
Asymptotes. Analysis of the behaviour of a function. Curvature, osculating circle.

   Taylor polynomial (special case: Maclaurin polynomial) of degree $n$ of a function $f$ at the point $x_0$. Derivation of formulas for the coefficients of the Taylor polynomial. Taylor's theorem. Lagrange form of the remainder and its use in problems. Approximation of functions by Taylor polynomials.

9. **week (16. 11. – 20. 11.) Fundamentals of integral calculus of functions of one real variable**
Antiderivative, indefinite integral. Sufficient condition for the existence of an antiderivative and an indefinite integral on an interval. Basic (tabulated) indefinite integrals.

   Integration by parts theorem. Applications to examples. Integration by substitution theorem. Applications of this theorem (the so-called first and second substitution methods). Calculation of indefinite

integrals using various substitutions. Integration of functions of the type $\sin^m x \cdot \cos^n x$.

10. **week (23. 11. – 27. 11.)**
Rational functions, decomposition into a sum of partial fractions. Integration of a rational function with a polynomial of degree at most 3 in the denominator.

Integration of rational functions of the type $R\left(x, \sqrt[n]{\dfrac{ax+b}{cx+d}}\right)$.

11. **week (30. 11. – 4. 12.)**
Riemann integral, geometric and physical interpretation, basic properties. Newton–Leibniz formula. Mean value of a function on an interval.

Integration by parts for the Riemann integral. Substitution method for the Riemann integral.

12. **week (7. 12. – 11. 12.)**
Applications of the Riemann integral: area under the graph of a function, volume of a solid generated by rotating the graph of a function, length of a curve (graph of a function).

Riemann integral as a function of the upper limit. Improper Riemann integral.

13. **week (14. 12. – 18. 12.)**
Numerical calculation of the Riemann integral, trapezoidal rule.

14. **week (4. 1. – 6. 1.)**
Replacement week: 4 January 2027 follows the even-week Monday schedule (replacement for 28 September 2026), 5 January 2027 follows the odd-week Tuesday schedule (replacement for 17 November 2026), and 6 January 2027 follows the even-week Wednesday schedule (replacement for 28 October 2026).