$$h\begin{pmatrix} 1 & 9 & 0 \\ 2 & 5 & 1 \end{pmatrix} = 2$$

$$B = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 5 & 1 \\ 1 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & 6 & 0 \\ 2 & 5 & 1 \\ 2 & 4 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & 6 & 0 \\ 0 & -1 & 1 \\ 2 & 4 & 2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 2 & 6 & 0 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & 6 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 6 & 0 \\ 0 & -1 & 1 \end{pmatrix} \Rightarrow h(B) = 2$$

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 2 \\ 5 & 4 & 1 \end{pmatrix}$$

1) $A = (a)$ 1×1 $\Rightarrow \det A = a$

2) 2×2: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow a.d - b.c$

3) 3×3: $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

$$\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = aei + dhc + gbf - gec - fha - ibd$$

$$\begin{matrix} a & b & c \\ d & e & f \end{matrix}$$

Sarrusovo prav.

4) 4×4 a více:

$$\begin{vmatrix} 1 & 3 & 2 & 5 \\ 0 & 1 & -1 & 3 \\ 2 & 1 & 2 & 0 \\ 3 & 1 & 1 & 0 \end{vmatrix} = \underbrace{1\cdot(-1)^{1+1}\begin{vmatrix} 1 & -1 & 3 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{vmatrix}}_{\text{alg. doplněk } a_{11}\ A_{11}} + 3(-1)^{1+2}\begin{vmatrix} 0 & -1 & 3 \\ 2 & 2 & 0 \\ 3 & 1 & 0 \end{vmatrix} + 2(-1)^{1+3}\begin{vmatrix} 0 & 1 & 3 \\ 2 & 1 & 0 \\ 3 & 1 & 0 \end{vmatrix} + 5(-1)^{1+4}\begin{vmatrix} 0 & 1 & -1 \\ 2 & 1 & 2 \\ 3 & 1 & 1 \end{vmatrix} =$$

$$= -3 - 3\cdot(-12) + 2\cdot(-3) - 5\cdot 5 = -3 + 36 - 6 - 25 = \underline{\underline{2}}$$

$$\begin{vmatrix} 1 & 3 & 2 & 5 \\ 0 & 1 & -1 & 3 \\ 2 & 1 & 2 & 0 \\ 3 & 1 & 1 & 0 \end{vmatrix} = 5\cdot(-1)^{1+4}\begin{vmatrix} 0 & 1 & -1 \\ 2 & 1 & 2 \\ 3 & 1 & 1 \end{vmatrix} + 3(-1)^{2+4}\begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 3 & 1 & 1 \end{vmatrix} + 0 + 0$$

$$= -5\cdot 5 + 3\cdot 9 = -25 + 27 = \underline{2}$$

$$\begin{vmatrix} 1 & 3 & 2 & 5 \\ 0 & 1 & -1 & 3 \\ 0 & -5 & -2 & -10 \\ 0 & -8 & -5 & -15 \end{vmatrix} = 1\begin{vmatrix} 1 & -1 & 3 \\ -5 & -2 & -10 \\ -8 & -5 & -15 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 3 \\ 0 & -7 & 5 \\ 0 & -13 & 9 \end{vmatrix} = 1\begin{vmatrix} -7 & 5 \\ -13 & 9 \end{vmatrix} =$$

$$= (-7)\cdot 9 - 5(-13) = -63 + 65 = \underline{\underline{2}}$$

$$\begin{vmatrix} \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & \cdot \end{vmatrix}$$

$k \in \mathbb{R}$ $\qquad k \neq 0 \Rightarrow k \cdot l = 1$

$$l = \frac{1}{k} = k^{-1}$$

$A \ (n \times n)$, $\det A \neq 0 \Rightarrow \exists\, A^{-1}:$

$$\underline{A \cdot A^{-1} = E}$$

$$A = \begin{pmatrix} 1 & 5 & -1 \\ 0 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 5 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & -4 & 4 & -1 & 0 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 5 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 12 & -1 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & -1 & 1 & 0 & 0 \\ 0 & 6 & 0 & 1 & 2 & -1 \\ 0 & 0 & 12 & -1 & 4 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 12 & 10 & 0 & 11 & 4 & 1 \\ 0 & 6 & 0 & 1 & 2 & -1 \\ 0 & 0 & 12 & -1 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 12 & 0 & 0 & 1 & -16 & 11 \\ 0 & 6 & 0 & 1 & 2 & -1 \\ 0 & 0 & 12 & -1 & 4 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{12} & -\frac{16}{12} & \frac{11}{12} \\ 0 & 1 & 0 & \frac{1}{6} & \frac{2}{6} & -\frac{1}{6} \\ 0 & 0 & 1 & -\frac{1}{12} & \frac{4}{12} & \frac{1}{12} \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} \frac{1}{12} & -\frac{16}{12} & \frac{11}{12} \\ \frac{1}{6} & \frac{2}{6} & -\frac{1}{6} \\ -\frac{1}{12} & \frac{4}{12} & \frac{1}{12} \end{pmatrix} = \frac{1}{12}\begin{pmatrix} 1 & -16 & 11 \\ 2 & 4 & -2 \\ -1 & 4 & 1 \end{pmatrix}$$

Zk:

$$\underbrace{\begin{pmatrix} 1 & 5 & -1 \\ 0 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}}_{A} \cdot \underbrace{\frac{1}{12}\begin{pmatrix} 1 & -16 & 11 \\ 2 & 4 & -2 \\ -1 & 4 & 1 \end{pmatrix}}_{A^{-1}} = \frac{1}{12}\begin{pmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

O.K.

$$A = \begin{pmatrix} \overset{+}{1} & \overset{-}{5} & \overset{+}{-1} \\ \overset{-}{0} & \overset{+}{1} & \overset{-}{2} \\ \overset{+}{1} & \overset{-}{1} & \overset{+}{3} \end{pmatrix} \qquad \det A = 13 - 1 = \underline{\underline{12}}$$

$$A^{-1} = \frac{1}{12}\begin{pmatrix} 1 & 2 & -1 \\ -16 & 4 & 4 \\ 11 & -2 & 1 \end{pmatrix}^{T} = \frac{1}{12}\begin{pmatrix} 1 & -16 & 11 \\ 2 & 4 & -2 \\ -1 & 4 & 1 \end{pmatrix}$$

$$X \cdot A = B \quad / \cdot A^{-1}$$

$$\boxed{\begin{aligned} X \cdot \underbrace{A \cdot A^{-1}}_{I} &= B \cdot A^{-1} \\ X &= B \cdot A^{-1} \end{aligned}}$$

$A \ldots n \times n$

$X .$

$B \ldots k \times n$

A^{-1}

$X \cdot I = X$

$X \cdot A = B$

$\cancel{A^{-1} \cdot X \cdot A = A^{-1} B}$

$\cancel{X \cdot A \cdot A^{-1} = A^{-1} \cdot B}$

$$\begin{aligned} 3x - y + z &= 5 \\ x + 2y - z &= 3 \\ x \quad\; + 2z &= 0 \end{aligned}$$

$$\begin{pmatrix} 3 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix}$$

$$A \cdot \vec{x} = \vec{b}$$

$$\underbrace{A^{-1} \cdot A} \cdot \vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = A^{-1} \cdot \vec{b}$$

$(A|\vec{b}) =$ řádkové ekvivalentní úpravy:

$$= \left(\begin{array}{ccc|c} 3 & -1 & 1 & 5 \\ 1 & 2 & -1 & 3 \\ 1 & 0 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -1 & 1 & 5 \\ 0 & 7 & -4 & 4 \\ 0 & 1 & 5 & -5 \end{array}\right) \sim \underbrace{\left(\begin{array}{ccc|c} 3 & -1 & 1 & 5 \\ 0 & 7 & -4 & 4 \\ 0 & 0 & 39 & -39 \end{array}\right)}$$

$$\begin{aligned} 3x - y + z &= 5 \\ 7y - 4z &= 4 \\ 39z &= -39 \end{aligned}$$

$z = -1$

$7y + 4 = 4 \Rightarrow y = 0$

$3x - 1 = 5 \Rightarrow x = 2$

Zk:

$$\begin{aligned} 3x - y + z &= 5 \\ x + 2y - z &= 3 \\ x \quad + 2z &= 0 \end{aligned} \quad \begin{pmatrix} x = 2 \\ y = 0 \\ z = -1 \end{pmatrix} \Longrightarrow \begin{aligned} 6 - 0 - 1 &= 5 \ \checkmark \\ 2 + 0 + 1 &= 3 \ \checkmark \\ 2 \quad - 2 &= 0 \ \checkmark \end{aligned}$$

o.k.

Cramer:

$$\begin{pmatrix} 3 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix}$$

$$\Delta = \det A = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 0 & 2 \end{vmatrix} = 12 + 1 - 2 + 2 = 13$$

$$\begin{matrix} 3 & -1 & 1 \\ 1 & 2 & -1 \end{matrix}$$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 1 \\ 3 & 2 & -1 \\ 0 & 0 & 2 \end{vmatrix} = 26 \qquad \Delta_2 = \begin{vmatrix} 3 & 5 & 1 \\ 1 & 3 & -1 \\ 1 & 0 & 2 \end{vmatrix} = 18 - 5 - 3 - 10 = 0$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 5 \\ 1 & 2 & 3 \\ 1 & 0 & \end{vmatrix} = -13 \quad \Longrightarrow \quad x = \frac{\Delta_1}{\Delta} = \frac{26}{13} = 2$$

$y = \frac{\Delta_2}{\Delta} = \frac{0}{13} = 0$

$z = \frac{\Delta_3}{\Delta} = \frac{-13}{13} = -1$

$$A \cdot X = \lambda X$$

$(n \times 1)$ nulový vektor

$$AX - \lambda X = 0$$

$$AX - \lambda EX = 0$$

$$\underbrace{(A - \lambda E)}X = 0$$

$B \cdot X = 0 \Rightarrow$ má vždy řešení $X = 0$
nenulové řeš. má tehdy, když $h(B) < n$
musí být $\underline{\det(A - \lambda E) = 0}$!

Př: $A = \begin{pmatrix} 3 & 5 & 1 \\ 0 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ $\quad \det(A - \lambda E) = \begin{vmatrix} 3-\lambda & 5 & 1 \\ 0 & 2-\lambda & 1 \\ 1 & 2 & 1-\lambda \end{vmatrix} =$

$\begin{matrix} 3-\lambda & 5 & 1 \\ 0 & 2-\lambda & 1 \end{matrix}$

$= (3-\lambda)(2-\lambda)(1-\lambda) + 5 - (2-\lambda) - 2(3-\lambda) =$

$= \dots\dots = 0 \Rightarrow \lambda = ?$

obtížné

Vl. čísla:

$B = \begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} 5-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix} = (5-\lambda)(3-\lambda) - 2 =$

$= 15 - 8\lambda + \lambda^2 - 2 = \underline{\lambda^2 - 8\lambda + 13 = 0}$

$\lambda_{1,2} = \frac{8 \pm \sqrt{64 - 52}}{2} = \frac{8 \pm \sqrt{12}}{2} =$

$= 4 \pm \sqrt{3}$

Vl. vektory:

$A\cancel{X} = \lambda X \quad = \quad \underline{(A - \lambda E)X = 0}$

$\lambda = 4+\sqrt{3}$: $\begin{pmatrix} 5-4-\sqrt{3} & 1 \\ 2 & 3-4-\sqrt{3}\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$(a-b)(a+b) = a^2-b^2$

$\begin{pmatrix}1-\sqrt3 & 1\\ 2 & -1-\sqrt3\end{pmatrix} \cdot (1+\sqrt3) \sim \begin{pmatrix}-2 & 1+\sqrt3\\2 & -1-\sqrt3\end{pmatrix} \sim \begin{pmatrix}-2 & 1+\sqrt3 \\ \cancel{0} & \cancel{0}\end{pmatrix} \Rightarrow -2x_1+(1+\sqrt3)x_2=0$

$x_2 = 1$

$-2x_1+1+\sqrt3=0$

$x_1 = \frac{1+\sqrt3}{2}$

$X_1 = \begin{pmatrix}\frac{1+\sqrt3}{2}\\1\end{pmatrix}\cdot k, \quad k\neq 0$

$\lambda = 4-\sqrt3$: $\begin{pmatrix}5-4+\sqrt3 & 1\\2 & 3-4+\sqrt3\end{pmatrix} \sim \begin{pmatrix}1+\sqrt3 & 1\\ 2 & -1+\sqrt3\end{pmatrix}\cdot(1-\sqrt3) \sim$

$\begin{pmatrix}-2 & 1-\sqrt3\\ \cancel{2} & \cancel{-1+\sqrt3}\end{pmatrix} \Rightarrow -2x_1+(1-\sqrt3)x_2=0$

$x_2 = k \neq 0 \qquad x_1 = \frac{1-\sqrt3}{2}k, \quad k\neq 0$

$-2x_1 = (-1+\sqrt3)k$

$X_2 = \begin{pmatrix}\frac{1-\sqrt3}{2}\\1\end{pmatrix}k, \quad k\neq 0$

Zk = ...

$\begin{pmatrix}5&1\\2&3\end{pmatrix}\begin{pmatrix}\frac{1-\sqrt3}{2}\\1\end{pmatrix} = \begin{pmatrix}\frac52(1-\sqrt3)+1\\ 1-\sqrt3+3\end{pmatrix} =$ doma

$\left[= (4-\sqrt3)\begin{pmatrix}\frac{1-\sqrt3}{2}\\1\end{pmatrix}\right]$

Test M1 - Algebra. 17.10.24

$\vec{a} = (1\ 0\ p)$
$\vec{b} = (1\ -1\ 6)$
$\vec{c} = (p\ 0\ 1)$

$$\begin{vmatrix} 1 & 0 & p \\ 1 & -1 & 6 \\ p & 0 & 1 \end{vmatrix} = -1 + 0 + 0 + p^2 = -1 + p^2 =$$

$p^2 - 1 = (p+1)(p-1) = 0$

$\cancel{p = \pm 1}$ $\quad p \in \mathbb{R} \setminus \{-1, 1\}$

c) $p = 2$ $\quad \vec{d} = (-2, 5, 3)$ $\quad \vec{a} = (1, 0, 2), \vec{b} = (1, -1, 6), \vec{c} = (2, 0, 1)$

$$\boxed{\alpha \vec{a} + \beta \vec{b} + \gamma \vec{c} = \vec{d}}$$

$$\begin{cases} \alpha + \beta + 2\gamma = -2 \\ -\beta = 5 \\ 2\alpha + 6\beta + \gamma = 3 \end{cases}$$

$\underline{\beta = -5}$

$$\begin{cases} \alpha + 2\gamma = 3 \\ 2\alpha + \gamma = 33 \end{cases} \quad \alpha = 3 - 2\gamma$$

$6 - 4\gamma + \gamma = 33$
$-3\gamma = 27$
$\underline{\gamma = -9}$

$\underline{\alpha = 21}$

$$\underline{\vec{d} = 21\vec{a} - 5\vec{b} - 9\vec{c}}$$

$\boxed{2}$ b)

$$\begin{cases} 2x + y - z = 1 \\ x - y - z = 0 \\ x + 2y + pz = 1 \end{cases} \quad \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & -1 & -1 & 0 \\ 1 & 2 & p & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -3 & -1 & -1 \\ 0 & 3 & 2p+1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -3 & -1 & -1 \\ 0 & 0 & 2p & 0 \end{array}\right) \Rightarrow$$

$p = 0 \Rightarrow \infty$ rez.
$p \neq 0 \Rightarrow r(A) = 3$
$\Rightarrow 1$ rez.

c) $p=1$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = -2 - 1 - 1 - 1 + 4 - 1 = \underline{-3}$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = -1 - 1 + 1 + 2 = -1$$

$$\Delta_y = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix} = -1 - 1 + 2 - 1 = -1$$

$\Rightarrow$

$x = \dfrac{\Delta_x}{\Delta} = \dfrac{1}{3}$

$y = \dfrac{1}{3}$

$$\Delta_z = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = -2+2+1-1 = 0$$

$$z = 0$$

$\boxed{1}$ b) $\vec{a} = (1, 1+\alpha, 1)$
$\vec{b} = (1-\alpha, 2, 0)$
$\vec{c} = (1, 2, \alpha)$

$$\begin{pmatrix} 1 & 1+\alpha & 1 \\ 1-\alpha & 2 & 0 \\ 1 & 2 & \alpha \end{pmatrix} \sim \begin{pmatrix} 1 & 1+\alpha & 1 \\ 0 & 1+\alpha^2 & -1+\alpha \\ 1 & 2 & \alpha \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 1+\alpha & 1 \\ 0 & 1+\alpha^2 & -1+\alpha \\ 0 & 1-\alpha & \alpha-1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1+\alpha & 1 \\ 0 & 1+\alpha^2 & -1+\alpha \\ 0 & -\alpha-\alpha^2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1+\alpha \\ 0 & \alpha-1 & 1+\alpha^2 \\ 0 & 0 & -\alpha(1+\alpha) \end{pmatrix}$$

$h(A) = 3 \iff -\alpha(1+\alpha) \neq 0 \iff \alpha \neq 0 \vee \alpha \neq -1$

Vektory budou LZ $\iff \alpha = 0 \wedge \alpha = -1$

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c) $\vec{z} = (3, -6, 4)$; $\vec{a} = (1, 3, 1)$, $\vec{b} = (-1, 2, 0)$; $\vec{c} = (1, 2, 2)$

$x\vec{a} + y\vec{b} + z\vec{c} = \vec{z}$

$$\left.\begin{aligned} 1x - 1y + 1z &= 3 \\ 3x + 2y + 2z &= -6 \\ 1x + 0y + 2z &= 4 \end{aligned}\right\}$$

$\boxed{2}$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix} \quad \det A = 4 \cdot \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -8 \neq 0 \Rightarrow \exists A^{-1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right) \begin{matrix} \cdot 2 \\ \\ \cdot 3 \end{matrix} \sim \Bigg($$

$$\left(\begin{array}{ccc|ccc} 2 & 4 & 0 & -1 & 0 & 3 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & -1 & -1 & 3 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} \end{array}\right) \Rightarrow A^{-1} = \frac{1}{4}\begin{pmatrix} -2 & -2 & 6 \\ 0 & 1 & 0 \\ 2 & 0 & -2 \end{pmatrix}$$

Zk:
$$A \cdot A^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix}\begin{pmatrix} -2 & -2 & 6 \\ 0 & 1 & 0 \\ 2 & 0 & -2 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \checkmark$$

c) $\det(A - \lambda E) = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 0 \\ 1 & 2 & 1-\lambda \end{vmatrix} = (1-\lambda)^2(4-\lambda) - 3(4-\lambda) =$$

$$= (4-\lambda)\left[(1-\lambda)^2 - 3\right] =$$

$$= (4-\lambda)(\lambda^2 - 2\lambda - 2) \stackrel{!}{=} 0 \Rightarrow$$

$$\lambda_1 = 4$$

$$\lambda_{2,3} = \frac{2 \pm \sqrt{4+8}}{2}$$

$$\lambda_{2,3} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

$\lambda_1 = 4$: $\quad (A - \lambda E)\vec{v} = \vec{0}$

$$\begin{pmatrix} -3 & 2 & 3 \\ \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & 2 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-3v_1 + 2v_2 + 3v_3 = 0$$
$$v_1 + 2v_2 - 3v_3 = 0$$
$$-2v_1 + 4v_2 = 0$$

$$v_2 = k \neq 0 \Rightarrow v_1 = 2v_2 = 2k$$

$$2k + 2k - 3v_3 = 0$$
$$v_3 = \frac{4k}{3}$$

$$\vec{v} = \begin{pmatrix} 2k \\ k \\ \frac{4}{3}k \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4/3 \end{pmatrix} k; \quad k \neq 0$$

Zk: $\quad A \cdot \vec{v} \stackrel{?}{=} \lambda \vec{v}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 4/3 \end{pmatrix} = \ldots$$