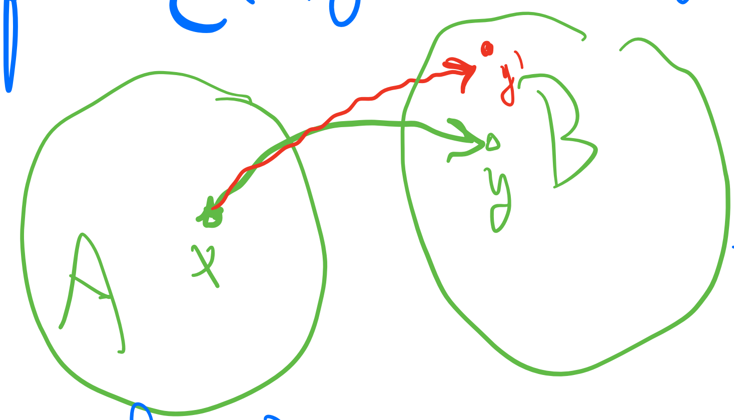
$f = \{(x,y)\in A\times B : \forall x\in A\ \exists! y\in B\}$



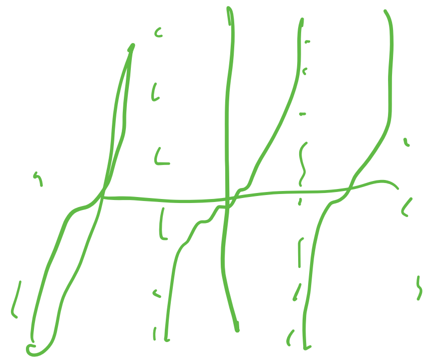
$A,B\subset\mathbb{R}$

$y=f(x)$

$A=D(f)$ $B=H(f)$

$f: (A,B,f)$

$D(f)=?$



$y=\frac{3+x}{2x}$, $x\in\langle 1,5\rangle$

$D(f)=\mathbb{R}-\{0\}=(-\infty;0)\cup(0;+\infty)$

$y=tg(x)$; $D(f)=\{x\in\mathbb{R}: x\neq\frac{\pi}{2}+k\pi, k\in\mathbb{Z}\}$

$D(f)\subset\mathbb{N} \Longrightarrow$ posloupnost

$D(f)\subset\mathbb{R} \Longrightarrow$ funkce reálné proměnné

$f(n)=\frac{n^2+2}{3n}$, $n\in\mathbb{N}$

$\{1;1;\frac{11}{9};\frac{18}{12};\dots\dots\}$ $+\infty$?

Pr. 1) $a_n = (-1)^n \quad \{-1; 1; -1; 1; -1; \dots\}$

2) $f(n) = \dfrac{n^2+2}{3n} \qquad \lim_{n\to\infty} \dfrac{n^2+2}{3n} = \lim_{x\to\infty} \dfrac{n(n+\frac{2}{n})}{n \cdot 3} =$

$n^2+2 \to +\infty, \quad 3n \to +\infty$

$\left\{\frac{1}{n}\right\} \longrightarrow 0$

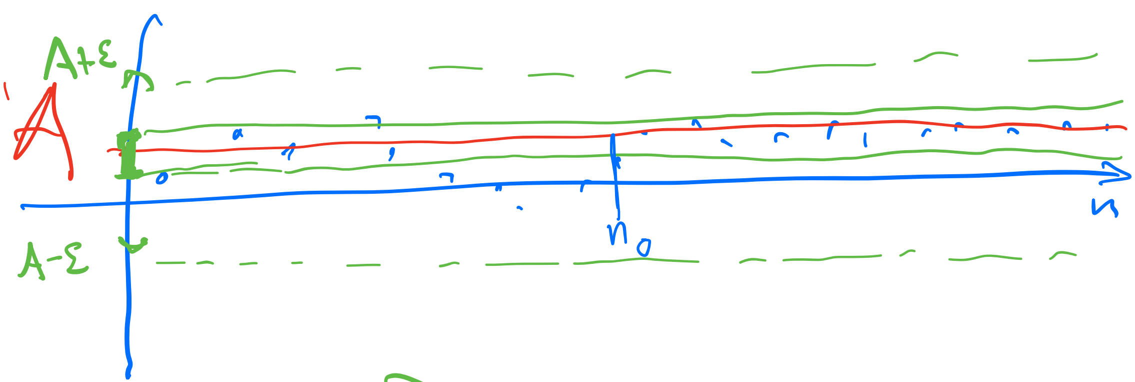
$= \lim_{n\to\infty} \dfrac{n+\frac{2}{n}}{3} = \infty$

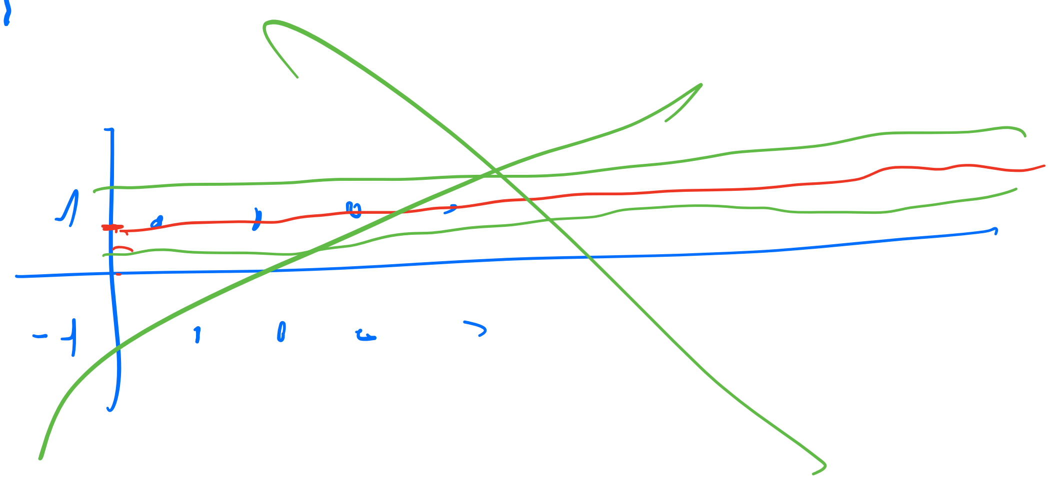
3) $f_n = \dfrac{n^2+2}{3n^2}; \quad \lim_{n\to\infty} \dfrac{n^2+2}{3n^2} = \lim_{n\to\infty} \dfrac{n^2(1+\frac{2}{n^2})}{n^2 \cdot 3} =$

$= \dfrac{1}{3}$

$A = \lim_{n\to\infty} a_n \iff \forall \varepsilon > 0 \ \exists n_0 : \forall n > n_0$

$|a_n - A| < \varepsilon$



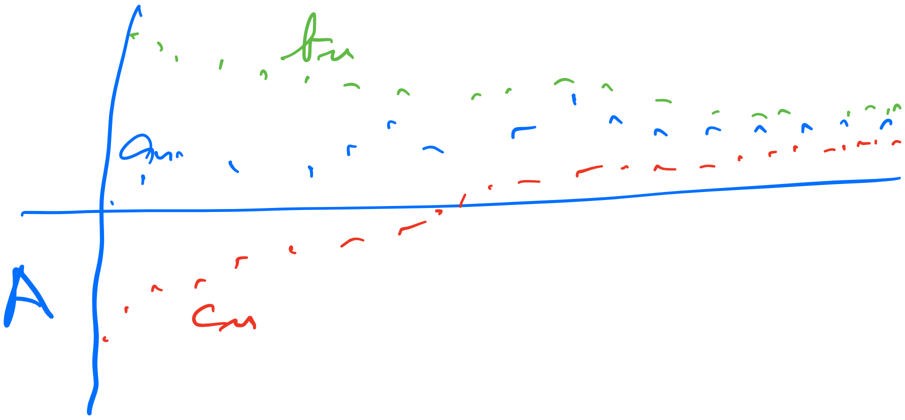


Věta o sevření posl.:

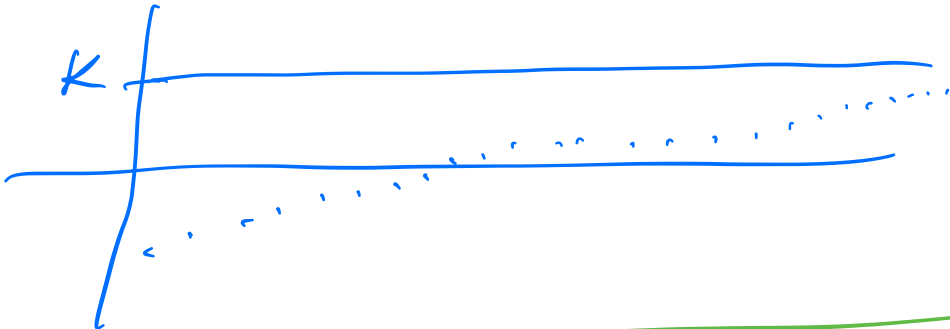
$c_n \le a_n \le b_n$

$A = \lim c_n = \lim b_n$

$\implies \exists \lim a_n = A$



Věta o limitě monotónní posl.



$y = f(x)$ $\quad D(f); H(f) \subset \mathbb{R}$

reálná fce reálné proměnné

$D(f) \subset \mathbb{R}; \; H(f) \subset \mathbb{C}$

komplexní fce reálné prom.

$D(f) \subset \mathbb{C}; \; H(f) \subset \mathbb{C}$

komplexní fce komplexní prom.

vnitřní fce

$z = x^2$

$y = \underbrace{\sin(x^2)}_{\text{vnější fce}}$ $\quad \sin(z)$

Př.: $$y = \frac{x^2+5}{\ln x} = \frac{A}{B} = \frac{z+5}{\ln x} =$$

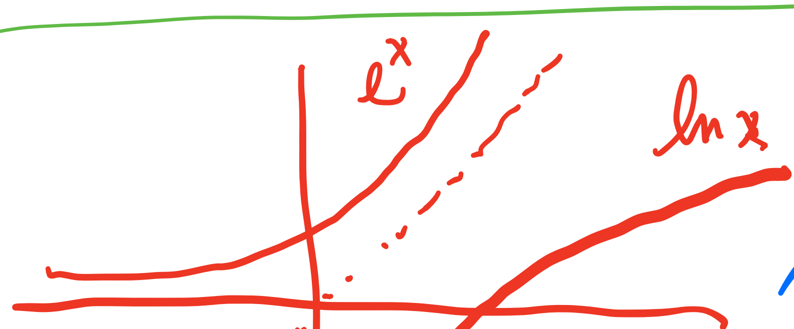
$$= \frac{x^2+5}{\ln x}$$

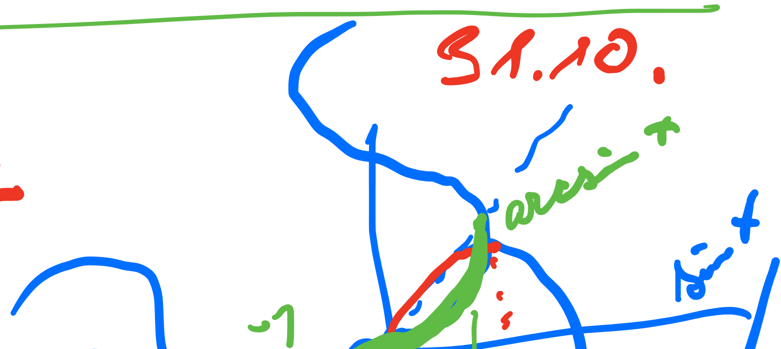
$y = \operatorname{tg}(x^2+1)$ vn. fce: $\operatorname{tg} z$

vnitř. fce: $z = x^2+1$

Elementární funkce:

1) $y = \text{const.}$
2) $y = \operatorname{sgn} x = \begin{cases} +1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$
3) $y = x^n$; $y = a_0 + a_1 x + a_2 x^2 + \dots a_n x^n$
4) $y = x^\alpha$, $x > 0$, $\alpha \in \mathbb{R}$
5) $y = \ln x$, $y = \log x$; $y = \log_a x$
6) $y = e^x$; $y = 10^x$; $y = a^x$
7) $y = \sin x$; $y = \cos x$; $y = \operatorname{tg} x$; $y = \operatorname{cotg} x$
8) $y = \arcsin x$; $y = \arccos x$; $y = \operatorname{arctg} x$; $y = \operatorname{arccotg} x$



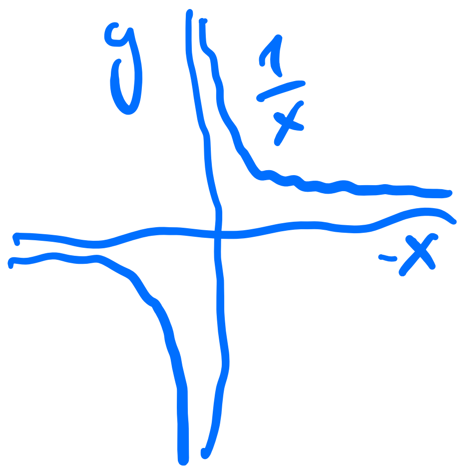


$$\lim_{x\to 1} \frac{e^x-1}{x} = \frac{e-1}{1} = \underline{\underline{e-1}}$$

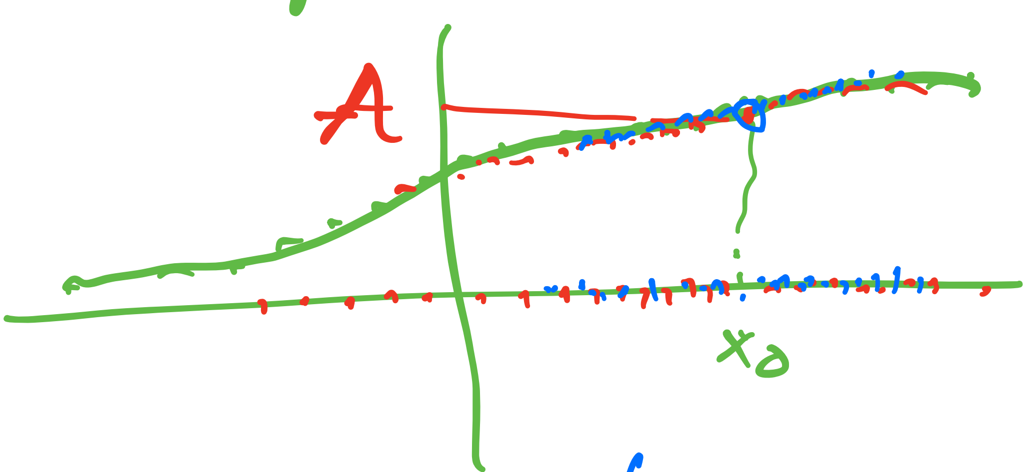
$$\lim_{x\to 0} \frac{e^x-1}{x} = \lim_{x\to 0} \underbrace{\frac{e^x}{x}}_{\infty} - \underbrace{\frac{1}{x}}_{\pm\infty} = \underline{\infty - \infty}$$

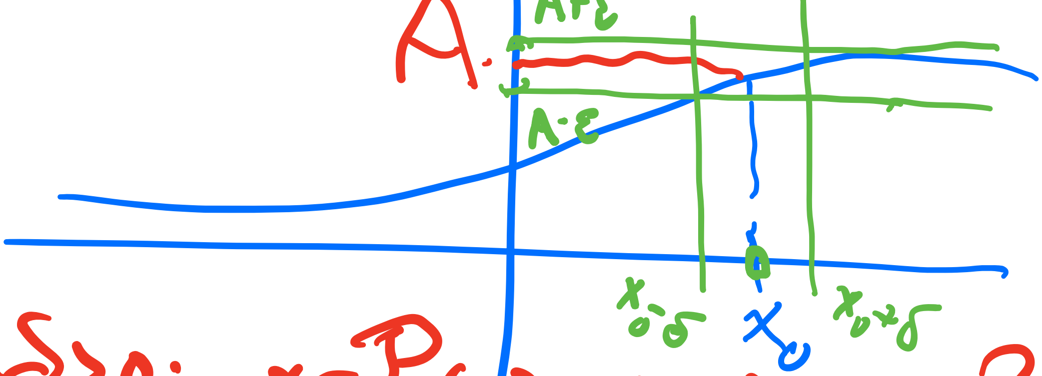
$$\Downarrow$$

$$\frac{\text{"}0\text{"}}{\text{"}0\text{"}} \quad [=1]$$

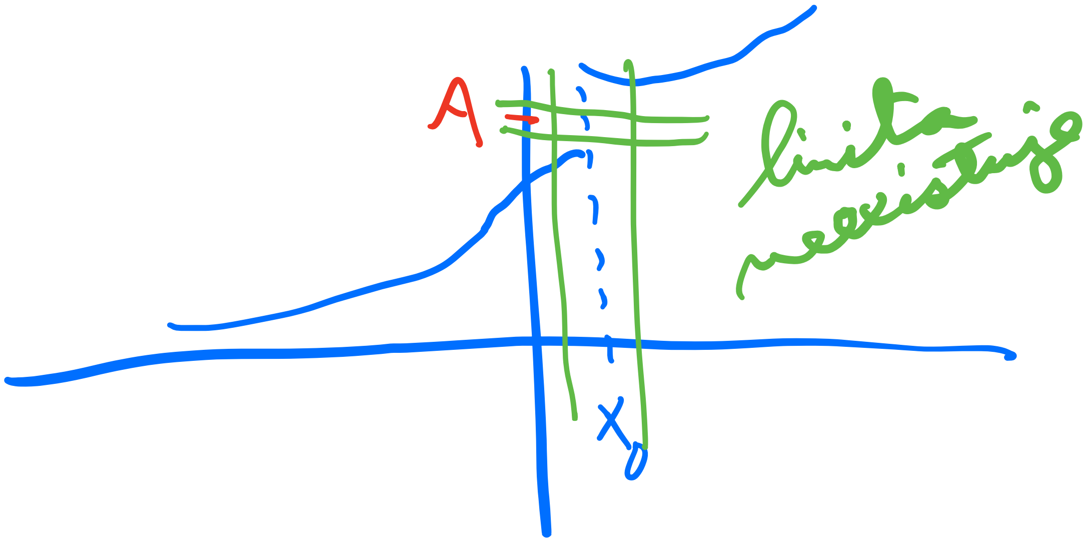


Limita funkce:





$$\forall \varepsilon>0\ \exists \delta>0:\ x\in P_\delta(x_0) \Rightarrow f(x)\in \mathcal{U}_\varepsilon(A)$$

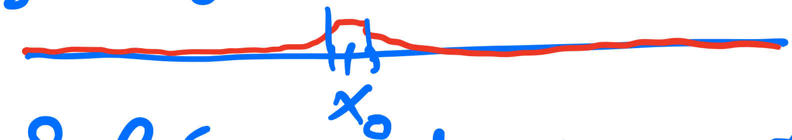


Rozlišujte mezi tzv. „lokální vlastnosti" a „globální vlastnosti".

Např.: Spojitost fce v bodě x_0 je lokální vlastnost. Vedle toho definujeme spojitost fce na intervalu – a ta je vlastnost globální.

Darboux:

Je-li $f(x)$ spojitá v x_0 a je $f(x_0) > 0 \Rightarrow$ ex.



okolí x_0: $f(x) > 0$ pro x z takového okolí.