$h\begin{pmatrix}1&9&0\\2&5&1\end{pmatrix} = 2$

$B = \begin{pmatrix}1&3&0\\2&5&1\\1&2&1\end{pmatrix} \sim \begin{pmatrix}2&6&0\\2&5&1\\2&4&2\end{pmatrix} \sim \begin{pmatrix}2&6&0\\0&-1&1\\2&4&2\end{pmatrix} \sim$

$\sim \begin{pmatrix}2&6&0\\0&-1&1\\0&-2&2\end{pmatrix} \sim \begin{pmatrix}2&6&0\\0&-1&1\\0&0&0\end{pmatrix} \sim \begin{pmatrix}2&6&0\\0&-1&1\end{pmatrix} \Rightarrow h(B) = 2$

$A = \begin{pmatrix}1&3&2\\0&1&2\\5&4&1\end{pmatrix}$

1) $A = (a)$ $1\times 1 \Rightarrow \det A = a$

2) 2×2: $A = \begin{pmatrix}a&b\\c&d\end{pmatrix} \Rightarrow a.d - b.c$

3) 3×3: $\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}$

$\det\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\\a&b&c\\d&e&f\end{pmatrix} = aei + dhc + gbf - gec - fha - ibd$

Sarrusovo prav.

4) 4×4 a více:

$\begin{vmatrix}1&3&2&5\\0&1&-1&3\\2&1&2&0\\3&1&1&0\end{vmatrix} = \underbrace{1\cdot(-1)^{1+1}\begin{vmatrix}1&-1&3\\1&2&0\\1&1&0\end{vmatrix}}_{\text{alg. doplněk } a_{11}\ A_{11}} + 3(-1)^{1+2}\begin{vmatrix}0&-1&3\\2&2&0\\3&1&0\end{vmatrix} + 2(-1)^{1+3}\begin{vmatrix}0&1&3\\2&1&0\\3&1&0\end{vmatrix} + 5(-1)^{1+4}\begin{vmatrix}0&1&-1\\2&1&2\\3&1&1\end{vmatrix} =$

$= -3 - 3.(-12) + 2\cdot(-3) - 5.5 = -3 + 36 - 6 - 25 = \underline{\underline{2}}$

$\begin{vmatrix}1&3&2&5\\0&1&-1&3\\2&1&2&0\\3&1&1&0\end{vmatrix} = 5.(-1)^{1+4}\begin{vmatrix}0&1&-1\\2&1&2\\3&1&1\end{vmatrix} + 3(-1)^{2+4}\begin{vmatrix}1&3&2\\2&1&2\\3&1&1\end{vmatrix} + 0 + 0$

$= -5\cdot 5 + 3\cdot 9 = -25 + 27 = \underline{2}$

$$\begin{vmatrix} 1 & 3 & 2 & 5 \\ 0 & 1 & -1 & 3 \\ 0 & -5 & -2 & -10 \\ 0 & -8 & -5 & -15 \end{vmatrix} = 1 \begin{vmatrix} 1 & -1 & 3 \\ -5 & -2 & -10 \\ -8 & -5 & -15 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 3 \\ 0 & -7 & 5 \\ 0 & -13 & 9 \end{vmatrix} = 1 \begin{vmatrix} -7 & 5 \\ -13 & 9 \end{vmatrix} =$$

$$= (-7)\cdot 9 - 5(-13) = -63 + 65 = \underline{\underline{2}}$$

$$\begin{vmatrix} & & & \\ 0 & & & \\ 0 & 0 & & \\ 0 & 0 & 0 & \end{vmatrix}$$

$k \in \mathbb{R}$ $\qquad k \neq 0 \Rightarrow k \cdot l = 1$

$$l = \frac{1}{k} = k^{-1}$$

A $(n \times n)$, $\det A \neq 0 \Rightarrow \exists\, A^{-1}:$

$$\underline{A \cdot A^{-1} = E}$$

$$A = \begin{pmatrix} 1 & 5 & -1 \\ 0 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 5 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & -4 & 4 & -1 & 0 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 5 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 12 & -1 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & -1 & 1 & 0 & 0 \\ 0 & 6 & 0 & 1 & 2 & -1 \\ 0 & 0 & 12 & -1 & 4 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 12 & 60 & 0 & 11 & 4 & 1 \\ 0 & 6 & 0 & 1 & 2 & -1 \\ 0 & 0 & 12 & -1 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 12 & 0 & 0 & 1 & -16 & 11 \\ 0 & 6 & 0 & 1 & 2 & -1 \\ 0 & 0 & 12 & -1 & 4 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{12} & -\frac{16}{12} & \frac{11}{12} \\ 0 & 1 & 0 & \frac{1}{6} & \frac{2}{6} & -\frac{1}{6} \\ 0 & 0 & 1 & -\frac{1}{12} & \frac{4}{12} & \frac{1}{12} \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} \frac{1}{12} & -\frac{16}{12} & \frac{11}{12} \\ \frac{1}{6} & \frac{2}{6} & -\frac{1}{6} \\ -\frac{1}{12} & \frac{4}{12} & \frac{1}{12} \end{pmatrix} = \frac{1}{12}\begin{pmatrix} 1 & -16 & 11 \\ 2 & 4 & -2 \\ -1 & 4 & 1 \end{pmatrix}$$

Zk:

$$\underbrace{\begin{pmatrix} 1 & 5 & -1 \\ 0 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}}_{A} \cdot \underbrace{\frac{1}{12}\begin{pmatrix} 1 & -16 & 11 \\ 2 & 4 & -2 \\ -1 & 4 & 1 \end{pmatrix}}_{A^{-1}} = \frac{1}{12}\begin{pmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

O.K.

$$A = \begin{pmatrix} {}^{+}1 & {}^{-}5 & {}^{+}-1 \\ {}^{-}0 & {}^{+}1 & {}^{-}2 \\ {}^{+}1 & {}^{-}1 & {}^{+}3 \end{pmatrix} \qquad \det A = 13 - 1 = \underline{\underline{12}}$$

$$A^{-1} = \frac{1}{12}\begin{pmatrix} 1 & 2 & -1 \\ -16 & 4 & 4 \\ 11 & -2 & 1 \end{pmatrix}^{T} = \frac{1}{12}\begin{pmatrix} 1 & -16 & 11 \\ 2 & 4 & -2 \\ -1 & 4 & 1 \end{pmatrix}$$