Mathematics II – Examples

III.4. Fubini's Theorem for triple integral

- Compute the following triple integral on given region $W \subset \mathbb{E}_3$:

Example 342: $I = \iiint_W (x^2 + y^2) \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$;
$$W = \{[x, y, z] \in \mathbb{E}_3 : \ 0 \le z \le x + y, \ 0 \le y \le 3, \ 0 \le x \le 2 \}.$$

Example 343: $I = \iiint_W \frac{x}{y}(z + 1)^2 \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$;
$$W = \{[x, y, z] \in \mathbb{E}_3 : \ 0 \le x \le 1, \ 1 \le y \le \mathrm{e}^2, \ 0 \le z \le 2 \}.$$

Example 344*: $I = \iiint_W z^3 y \sin x \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$;
$$W = \{[x, y, z] \in \mathbb{E}_3 : \ 0 \le z \le \sin x, \ 0 \le y \le \sin^2 x, \ 0 \le x \le \tfrac{\pi}{2} \}.$$