Numerical mathematics

NUMERICAL MATHEMATICS

Summer semester 2024/2025

Lectures: Thursday 12:30-14:00 (room T4:C2-438)
Tutorials: Friday 12:30-14:00 (room T4:A1-405a)

Course Schedule

Week 1

- Introduction.
- Fixed point iterations: in R with graphical illustration, in R^n - introduction.
- Vector norms - illustration.
- Matrices - what are they? Video (I recommend the whole course on linear algebra).
  Graphically: 2x2 matrices as linear transformations
- Matrix properties. Graphically: Spectral norm and spectral radius.
- Illustrations in Matlab:
  - Fixed point iterations
  - Norms and other properties of matrices
  - Matrices as 2D transformations - graphically
  - Matrix norms - graphically
- HW

Week 2

- fixed point iterations in R^n
- Iterative methods for linear systems:
  - graphically: powers of a matrix, Jacobi and Gauss-Seidel iterations
- Solved problems
- Illustrations in Matlab:
  - Powers of a matrix - graphically
  - Iterative methods for linear systems
- HW

Week 3

- Newton's method, example.
- Illustration in Matlab.
- Video: Newton's method in R and C, Newton's fractal
- Recapitulation.
- HW

Week 4

- Substitution of derivatives with finite differences.
- Cauchy problem for first-order ordinary differential equation:
  explicit and implicit Euler's method, midpoint (Collatz) method.
- Existence and uniqueness of exact solution: Example.
- Illustration in Matlab.
- Video: Euler's method - the basics
  Video: Midpoint method - the basics - first 6:30 minutes
- HW

Week 5

- Cauchy problem for systems of ODEs:
  explicit and implicit Euler's method, midpoint (Collatz) method.
- Explicit Runge-Kutta methods - example.
- One-step methods graphically
- Illustration in Matlab.
- Wiki - Runge-Kutta methods.
- HW

Week 6

- One-step methods - consistence, stability, convergence.
- Behavior of errors.
- Stability of Euler's methods: Illustration in Matlab.
- Video: Gilbert Strang (the first 25 minutes)
- Video, Gilbert Strang: Lecture I, Lecture II
- Recapitulation.
- HW

Week 7

- Boundary value problem for ordinary differential equations.
- Illustration in Matlab.
- Video: Gilbert Strang, MIT
- HW

Week 8

- Classification of the 2-nd order linear partial differential equations of two independent variables.
- Dirichlet problem for Poisson equation, Finite difference method.
- Illustration in Matlab.
- Video: Laplacian
- HW

Week 9 ... 21-st of April: Tutorial cancelled (Easter holiday)

- Mixed problem for heat equation, Finite difference method.
- Graphical illustration.
- Illustration in Matlab.
- Video: Heat equation

Week 10

- Mixed problem for wave equation, Finite difference method.
- Illustration in Matlab.

- HW from week 9

Week 11 ... 1-st of May: Lecture cancelled (holiday)

- HW from week 10

Week 12 ... 8-th of May: Lecture cancelled (holiday)

- Recapitulation.

Week 13

- Approximation by polynomials - the least squares method.
- Illustration in Matlab.
- Gradient methods. The steepest descent method.
  Illustration in Matlab.
- Video: Gilbert Strang (the least squares from approx. 25-th minute)
- HW
- Recapitulation.

Week 14

- **Assessment test:** Thursday 22-nd of May, 12:30 - 14:00, T4:C2-438

Exams: Requirements for exams. At exam, you should expect similar problems to those given as HWs together with theoretical questions like these, see also requirements above. Examples of exam tests.

References

- T. Petersdorff: Fixed Point Iteration and Contraction Mapping Theorem
- Y. Saad: Iterative methods for sparse linear systems (pdf)
- J. R. Chasnov: Numerical Methods for Engineers
- G. Strang: Computational Science and Engineering, selected chapters
- C. T. Kelley: Iterative Methods for Linear and Nonlinear Equations, SIAM 1995
- T. Petersdorff: Errors for Linear Systems
- M. Zeltkevic: Forward and Backward Euler Methods
- E. Cheever: Fourth Order Runge-Kutta
- I. Berg: Comparison of RK methods
- Joel Feldman: Variable Step Size Methods
- Matlab tutorial - Clarkson University - html
- * K. B. Petersen, M. S. Pedersen: The Matrix Cookbook - pdf

Video Lectures

- Linear transformations and matrices (video)
- 3Blue1Brown channel: Essence of linear algebra
- Gilbert Strang: Linear algebra, Unit II: Least Squares, Determinants and Eigenvalues
- Gilbert Strang: Computational Science and Engineering I, 2008
- Gilbert Strang: Computational Science and Engineering II, 2006
- 3BLUE1BROWN SERIES: Differential equations, studying the unsolvable | DE1

- Logistic equation - Video

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