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<!DOCTYPE html PUBLIC „-W3CDTD XHTML 1.0 StrictEN“
„http://www.w3.org/TR/2000/REC-xhtml1-20000126/DTD/xhtml1-strict.dtd“> <html>
<title>Numerical&nbsp;mathematics</title> </head><body> <div id=„container“> <div
id=„content“> <h1>NUMERICAL&nbsp;MATHEMATICS</h1> <p>Summer semester 2024/2025 </p>
<p> </p> <!-- <em class=„redbold“>Exam terms:</em> <ul> <li><s></s>Mon 20-th of May, from
9:00 in KN:A-309 </li> <li>Tue 28-th of May, from 9:00 (wait from 8:50 in front of KN:A-214, please)
</li> <li>Tue 4-th of June, from 9:00 (wait from 8:50 in front of KN:A-214, please) </li> <li>Tue 11-th
of June, from 9:00 (wait from 8:50 in front of KN:A-214, please) </li> <li>Tue 25-th of June, from 9:00
(wait from 8:50 in front of KN:A-214, please) </li> </ul> <p> For enrolment to exam, student must
have a <strong>valid assessment</strong> from tutorials, registered in the electronic system KOS.
For <strong>terms after 20-th of May</strong>, student has to <strong>register in the
KOS</strong> system for the chosen date of the exam. </p> <p> <strong>Caution:</strong> The
exam will take place at different classroom than the one specified in KOS - wait in front of KN: A-214
from 8:50, please, I will meet you there and tell you the exact place of the exam. </p> <p> <em
class=„redbold“>Assessment test term:</em> </p><ul> <li><s></s>Mon 13-th of May, from 9:00
in KN:A-309 </li> </ul> -> <p></p> <!-- </p-> <!--<hr><hr> <p> 2020: Exam test will be
<strong>available from 15:00 Czech time</strong>. Then you will have 5 minutes for reporting any
problems with opening the test<!-- (be careful to open the appropriate level - mostly B)>. The Lecture
meeting in MS Teams will be opened from 14:55 to the end of the test for reporting problems.
<br><br> <strong>Follow the instructions on the first page of the test, please.</strong> <br>
<hr-> <hr> <p> <s></s>
Lectures:&nbsp;&nbsp;&nbsp;Thursday&nbsp;&nbsp;&nbsp;12:30-14:00&nbsp;&nbsp;&nbsp;(room&nbsp;&nbsp;&nbsp;T4:C2-438)
<br> Tutorials:&nbsp;&nbsp;&nbsp;Friday&nbsp;&nbsp;&nbsp;12:30-14:00&nbsp;&nbsp;&nbsp;(room&nbsp;&nbsp;&nbsp;T4:A1-405a)
</p> <ul class=„biblio“> <!--<li><a href=„http://mat.fs.cvut.cz/numer“> Detailed information </a>
(in Czech) </li-> </ul> <hr> <h2>Course Schedule <!-- - Lectures -></h2> <em class=„redbold“>
Week 1</em><em></em> <ul> <li><a href=„NM/Alntro.pdf“>Introduction</a>.</li> <li>Fixed
point iterations:&nbsp;   <!--a
href=„http://home.iitk.ac.in/~psraj/mth101/lecture_notes/lecture8.pdf“>in  $R$ </a-> <a
href=„NM/AfixR1.pdf“>in  $R$ </a> with graphical illustration,&nbsp;   <a href=„NM/Afix.pdf“>in  $R^n$ </a> -
introduction.</li> <li>Vector norms - <a href=„NM/motiv_metrics.pdf“>illustration.</a></li>
<li>Matrices - what are they? <a
href=„https://www.youtube.com/watch?v=kYB8IZa5AuE&amp;list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab&amp;index=3&amp;t=47s&amp;ab_channel=3Blue1Brown“> Video</a> (I recommend the
whole <a
href=„https://www.youtube.com/playlist?list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab“>course on
linear algebra</a>).<br> Graphically: <a href=„NM/domek.pdf“> $2 \times 2$  matrices as linear
transformations</a> </li> <li><a href=„NM/Amatr.pdf“>Matrix properties.</a> Graphically: <a
href=„NM/Anorms.pdf“>Spectral norm and spectral radius</a>.</li> <li>Illustrations in Matlab:
<ul> <li><a href=„NM/Am_fix.html“>Fixed point iterations</a></li> <li><a
href=„NM/Amat1.html“>Norms and other properties of matrices</a></li> <li><a
href=„NM/Amat2.html“>Matrices as 2D transformations</a> - graphically</li> <li><a
href=„NM/norms.txt“>Matrix norms</a> - graphically</li> </ul> </li> <li> <a
href=„NM/Adu1.pdf“>HW</a></li> </ul> <em class=„redbold“> Week 2</em><em></em> <ul>
<li> <a href=„NM/Afix.pdf“>fixed point iterations in  $R^n$ </a> </li> <li> Iterative methods for linear
systems:<br> - graphically: <a href=„NM/Apowers.pdf“>powers of a matrix</a>, <a
href=„NM/jgs.html“>Jacobi and Gauss-Seidel iterations</a><br> </li> <li><a
href=„NM/Alin.pdf“>Solved problems</a></li> <li>Illustrations in Matlab: <ul> <li><a
href=„NM/Amat3.html“>Powers of a matrix</a> - graphically</li> <li><a
href=„NM/Am_lin.html“>Iterative methods for linear systems</a></li> </ul> </li> <li> <a
href=„NM/Adu2.pdf“>HW</a></li> </ul> <em class=„redbold“> Week 3</em><em></em> <ul>
<li> <a href=„NM/Anewt.pdf“>Newton's method</a>, <a href=„NM/Anewt_ex1.pdf“>example</a>.

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[Illustration in Matlab.](#) Video: https://www.youtube.com/watch?v=-RdOwhmqP5s&ab_channel=3Blue1Brown Newton's method in R and C, Newton's fractal

Recapitulation.

HW

Week 4

- Substitution of [derivatives with finite differences](#).
- [Cauchy problem for first-order ordinary differential equation:](#) explicit and implicit Euler's method, midpoint (Collatz) method.
- Existence and uniqueness of exact solution: [Example.](#)
- [Illustration in Matlab.](#) Video: https://www.youtube.com/watch?v=_0mvWedqW7c Euler's method - the basics
- Video: <https://www.youtube.com/watch?v=2hjoqAaH5kc> Midpoint method - the basics - first 6:30 minutes
- HW

Week 5

- [Cauchy problem for systems of ODEs:](#) explicit and implicit Euler's method, midpoint (Collatz) method.
- Explicit Runge-Kutta methods - [example](#).
- One-step methods [graphically](#)
- [Illustration in Matlab.](#)
- [Wiki - Runge-Kutta methods](https://en.wikipedia.org/wiki/Runge%E2%80%93Kutta_methods).
- HW

Week 6

- [One-step methods](#) - consistence, stability, convergence. [Order of methods,](#)
- [Behavior of errors.](#)
- Stability of Euler's methods: [Illustration in Matlab.](#) Video: <https://ocw.mit.edu/courses/mathematics/18-085-computational-science-and-engineering-i-fall-2008/video-lectures/lecture-10-finite-differences-in-time/> Gilbert Strang (the first 25 minutes)
- Video, Gilbert Strang: <http://www.infocobuild.com/education/audio-video-courses/mathematics/18-086-Mathematical-Methods-MITOCW/lecture-01.html>, <http://www.infocobuild.com/education/audio-video-courses/mathematics/18-086-Mathematical-Methods-MITOCW/lecture-02.html> Lecture II
- Recapitulation.
- HW

Week 7

- [Boundary value problem](#) for ordinary differential equations.
- [Illustration in Matlab.](#) Video: <https://ocw.mit.edu/courses/mathematics/18-085-computational-science-and-engineering-i-fall-2008/video-lectures/lecture-2-differential-eqns-and-difference-eqns/> Gilbert Strang, MIT
- HW

Week 8

- Substitution of [derivatives with finite differences](#) in 2D.
- [Classification](#) of the 2-nd order linear partial differential equations of two independent variables.
- Dirichlet problem for [Poisson equation](#), Finite difference method.
- [Illustration in Matlab.](#) Video: <https://www.youtube.com/watch?v=EW08rD-GFh0> Laplacian
- HW

Week 9

... 21-st of April: Tutorial cancelled (Easter holiday)

- Mixed problem for [heat equation](#), Finite difference method.
- [Graphical illustration.](#)
- [Illustration in Matlab.](#) Video: <https://www.youtube.com/watch?v=ly4S0oi3Yz8&t=141s> Heat equation
- HW

Week 10

- Mixed problem for [wave equation](#), Finite difference method.
- [Illustration in Matlab.](#)

[HW](„NM/Adu9.pdf“) from week 9

- Week 11** ... 1-st of May: Lecture cancelled (holiday)
 - [HW](„NM/Adu10.pdf“) from week 10
 - Week 12** ... 8-th of May: Lecture cancelled (holiday)
 - Recapitulation.
- Week 13**
 - [Approximation by polynomials](„NM/Aintap.pdf“) - the least squares method.
 - [Illustration in Matlab.](„NM/Am_app.html“)
 - Gradient methods.
 - [The steepest descent method](„NM/Amns.pdf“).

[Illustration in Matlab](„NM/am_grad.html“).
 - Video: [https://ocw.mit.edu/courses/mathematics/18-085-computational-science-and-engineering-i-fall-2008/video-lectures/lecture-10-finite-differences-in-time/](„https://ocw.mit.edu/courses/mathematics/18-085-computational-science-and-engineering-i-fall-2008/video-lectures/lecture-10-finite-differences-in-time/“) Gilbert Strang (the least squares from approx. 25-th minute)
 - [HW](„NM/Adu11.pdf“)
 - Recapitulation.
 - Recapitulation.
- [Examples of test problems](„NM/NM_ang_testex_B.pdf“) (B level).
 - Week 14**
 - Assessment test:**

 Thursday 22-nd of May, 12:30 - 14:00, T4:C2-438
 - Information:**
 - You should be present 10 minutes before the beginning of the test, so that I can give you some short instructions and you are prepared to start writing at 9:00.
 - No electronic devices are allowed (calculator, notebook, watch, ... etc.). You can use only a sheet of paper which you will be given and your ballpoint pen.
 - Results will be available on the next Tutorial (Thu 12-th of May at 9:00, room KN:A-447)
 - Results** of the test are [here](„NM/vysl.html“).

10:45 online

Exam tests will be available from 10:45 Czech time. Then you will have 5 minutes for reporting any problems with opening the test (be careful to open the appropriate level - mostly B). The Lecture meeting in MS Teams will be opened from 10:40 to the end of the test for reporting problems.

Follow the instructions on the first page of the test, please.
 - [Exam test, level A](„NM/NM_ang_21_5_13_A.pdf“)

[Exam test, level B](„NM/NM_ang_21_5_13_B.pdf“)
 - Start:** 10:50 Czech time

End: 12:35 Czech time (90 + 15 min)

Any solution sent after 12:35 Czech time will NOT BE ACCEPTED.

Exams: [Requirements](„NM/AA_ex.html“) for exams. At exam, you should expect similar problems to those given as HWs together with theoretical questions like [these](„NM/ANM_zk_th.pdf“), see also requirements above. [Examples](„NM/NM_ang_vzor.pdf“) of exam tests.

References

- T. Petersdorff: [Fixed Point Iteration and Contraction Mapping Theorem](„http://terpconnect.umd.edu/~petersd/666/fixpoint.pdf“)
- Y. Saad: [Iterative methods for sparse linear systems](„http://www-users.cs.umn.edu/~saad/books.html“) ([pdf](„http://www-users.cs.umn.edu/~saad/IterMethBook_2ndEd.pdf“))
- J. R. Chasnov: [Numerical Methods for Engineers](„https://www.math.hkust.edu.hk/~machas/numerical-methods-for-engineers.pdf“)
- G. Strang: Computational Science and Engineering, [selected chapters](„http://ocw.mit.edu/courses/mathematics/18-086-mathematical-methods-for-engineers-ii-spring-2006/readings/“)
- C. T. Kelley: [Iterative Methods for Linear and Nonlinear Equations](„http://www.siam.org/books/textbooks/fr16_book.pdf“), SIAM 1995
- T. Petersdorff: [Errors for Linear Systems](„http://terpconnect.umd.edu/~petersd/460/linsystemrn.pdf“)
- M. Zeltkevic: [http://web.mit.edu/10.001/Web/Course_Notes/Differential_Equations_Notes/node3.html](„http://web.mit.edu/10.001/Web/Course_Notes/Differential_Equations_Notes/node3.html“)

Forward and Backward Euler Methods

- E. Cheever: <http://ipsa.swarthmore.edu/NumInt/NumIntFourth.html> Fourth Order Runge-Kutta
- D. N. Arnold: <http://www.ima.umn.edu/~arnold/papers/stability.pdf> Stability, consistency, and convergence of numerical discretizations
- I. Berg: https://beltoforion.de/en/runge-kutta_vs_euler/index.php#idStart Comparison of RK methods
- Joel Feldman: <https://personal.math.ubc.ca/~feldman/math/vble.pdf> Variable Step Size Methods
- <http://www.cyclismo.org/tutorial/matlab/> Matlab tutorial - Clarkson University
- * K. B. Petersen, M. S. Pedersen: http://www2.imm.dtu.dk/pubdb/views/edoc_download.php/3274/pdf/imm3274.pdf The Matrix Cookbook - pdf
- <http://www.cems.uvm.edu/~tlakoba/math337/>, <http://www.mathematik.uni-dortmund.de/~kuzmin/cfdintro/lecture9.pdf>
- <http://www.ima.umn.edu/~arnold/8445-8446.14-15/notes.pdf>
- http://www.ann.jussieu.fr/frey/cours/UdC/ma691/ma691_ch6.pdf
- http://videolectures.net/mit18085f07_computational_science_engineering/
- http://www2.imm.dtu.dk/pubdb/views/edoc_download.php/3274/pdf/imm3274.pdf

Video Lectures

- https://www.youtube.com/watch?v=kYB8IZa5AuE&index=4&list=PLZHQObOWTQPD3MizzM2xvFitgF8hE_ab&t=0s Linear transformations and matrices (video)
- 3Blue1Brown channel: <https://goo.gl/R1kBdb> Essence of linear algebra
- Gilbert Strang: Linear algebra, <http://ocw.mit.edu/courses/mathematics/18-06sc-linear-algebra-fall-2011/least-squares-determinants-and-eigenvalues/> Unit II: Least Squares, Determinants and Eigenvalues
- Gilbert Strang: <http://ocw.mit.edu/courses/mathematics/18-085-computational-science-and-engineering-i-fall-2008/video-lectures/> Computational Science and Engineering I, 2008
- Gilbert Strang: <http://www.infocobuild.com/education/audio-video-courses/mathematics/18-086-Mathematical-Methods-MITOCW/lecture-01.html> Computational Science and Engineering II, 2006
- 3BLUE1BROWN SERIES: https://www.youtube.com/watch?v=p_di4Zn4wz4 Differential equations, studying the unsolvable | DE1
- Logistic equation - https://www.youtube.com/watch?v=ovJcsL7vyrk&ab_channel=Veritasium Video

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